WWCA 2016 Abstracts

Organized by Carsten Schneider, Eugene Zima
July 23-24, 2016, WLU, Waterloo, Canada

Desingularization of First Order Linear Difference Systems with Rational Function Coefficients

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It is well known that for a first order system of linear difference equations with rational function coefficients, a solution that is holomorphic in some left half plane can be analytically continued to a meromorphic solution in the whole complex plane. The poles stem from the singularities of the rational function coefficients of the system. Just as for systems of differential equations, not all of these singularities necessarily lead to poles in a solution, as they might be what is called removable. In this talk, we show how to detect and remove these singularities and further study the connection between poles of solutions, removable singularities and the extension of numerical sequences at these points. This is a joint work with Maximilian Jaroschek

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Bivariate Extensions of Abramov's Algorithm for Rational Summation

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Symbolic summation has been a powerful tool in combinatorics and mathematical physics, whose history is as long as that of symbolic computation. Abramov's algorithm for rational summation is one of the first few fundamental algorithms in symbolic summation. This paper is to present several extensions of Abramov's Algorithm for solving summation problems of bivariate rational functions.

RoadMap algorithm for constructing dessins d'enfants

Mark van Hoeij

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A dessin d'enfant is a connected, oriented, bi-colored graph. Up to equivalence, such graphs are in one-to-one correspondence with functions on Riemann surfaces that branch only above 0, 1, and infinity. This means that classifying such functions, called Belyi functions, can be done with combinatorial means.

The goal of this talk is to give an algorithm to find all dessins with a prescribed branching pattern. The basic strategy is, starting from the empty dessin, to keep inserting edges and vertices until one reaches the target dessins (dessins with the prescribed branching pattern). However, the number of dessins grows rapidly (oeis.org/A057005). In arXiv:1604.08158 this growth was reduced by discarding dessins that can not be extended to a target dessin. However, finding dessins of high degree N remained problematic. For each target dessin T of degree N there may be a huge number of dessins of intermediate degrees d < N that can be extended to T. But for each T and d we only need one!

To make the basic strategy of inserting edges and vertices more efficient, it needs a RoadMap – a description which intermediate dessins to extend, and which to discard. The goal of the RoadMap algorithm is to cut as many as possible (but not all!) of the paths from the empty dessin to the target dessins.

Asymptotic Log-convexity

Quing-Hu Hou

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We present a method of proving log-convexity based on the asymptotic behavior of the sequence. A sequences $\{a_n\}$ is said to be asymptotic log-convex if it is log-convex for n sufficient large. We show that for a kind of sequences, asymptotic log-convex implies asymptotic k-log-convex for any positive integer k. Moreover, we could find the bound N such that the sequence $\{a_n\}$ is log-convex for $n > N$. As an application, we show that a P-recursive sequence is asymptotic k-log-convex if and only if it is log-convex. By checking the initial values, we are able to prove the k-log-convexity of some well-known sequences for explicit k.

Factorization of C-finite Sequences

Manuel Kauers

Johannes Kepler University, Linz, Austria

We all know that the sum and the product of two C-finite sequences is again C-finite, and it is not hard to find recurrence equations for the sum or the product of two given C-finite sequences using linear algebra. Conversely, it is also not too difficult to decide whether a given C-finite sequence can be written nontrivially as a sum of two simpler C-finite sequences. This just requires the factorization of a univariate polynomial. In the talk, we will consider the analogous problem for the product: given a C-finite sequence, the task is to decide whether it can be written nontrivially as the product of two simpler C-finite sequences. An algorithm for solving this problem was first given by Ritt in the 1920s. We will present an alternative algorithm that is somewhat simpler and seemingly not less efficient than Ritt's approach, and we mention two applications to tiling problems and statistical mechanics.
This is joint work with Doron Zeilberger.

Inverse Inequality Estimates with Symbolic Computation

Christoph Koutschan

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In the convergence analysis of numerical methods for solving partial differential equations (such as finite element methods) one arrives at certain generalized eigenvalue problems, whose maximal eigenvalues need to be estimated as accurately as possible. With the help of computer algebra, in particular with symbolic summation techniques, we are able to improve a previously known upper bound by a factor of 8. More precisely, we aim at evaluating the corresponding determinant using the holonomic ansatz (Zeilberger, 2007), which is a powerful tool that translates the determinant problem into summation identities. However, it turns out that this method does not succeed on the problem at hand.

As a solution we present a variation of the original holonomic ansatz that is applicable to a larger class of determinants, including the one we are dealing with here. We obtain an explicit closed form for the determinant, whose special form enables us to derive new and tight upper resp. lower bounds on the maximal eigenvalue, as well as its asymptotic behavior.
This is joint work with Martin Neumueller and Cristian-Silviu Radu.

Abramov's Influence on Research at KLMM

Ziming Li

KLMM, AMMS,CAS, Beijing, China

The talk reviews two important results: one is by Abramov and Petkovšek on minimal decompositions of hypergeometric terms; the other is by Abramov on the applicability of Zeilberger's algorithm. We describe how the two results led to a number of publications co-authored by members at KLMM. We also mention an additive decomposition to decide the q-hypergeometric summability without solving any auxiliary q-recurrence equations.

Denominator Bounds for Higher Order Recurrence Systems over $\Pi - \Sigma$ Fields

Johannes Middeke

RISC, Johannes Kepler University Linz, Austria

We consider higher order systems of linear recurrence equations with coefficients in a $\Pi-\Sigma$ field. We are interested in computing rational solutions. A first step is to determine a so-called "denominator bound", ie, a polynomial that is a multiple of the denominator of all possible solutions of the system. We present a way how this can be computed directly from the higher order system without converting it to first order.

Representation of (q)-hypergeometric products and mixed versions in difference rings

Evans Doe Ocansey

RISC, Johannes Kepler University Linz, Austria

In this talk, we will present how one can algorithmically represent mixed hypergeometric products defined over a field K where K is a rational function field over an algebraic number field. More precisely, we will denomonstrate how one can solve the following problem:

Given a finite number of mixed hypergeometric products that evaluate to our constant field K; construct a difference ring extension where the constant field K might be extended by algebraic number extensions) and compute elements in the obtained difference ring such that they represent the given input products.

Based on difference ring theory, the produced representation of the above problem specification is given in terms of (q)-hypergeometric products and mixed versions that are algebraically independent among each other.

Furthermore, with this result, we obtain a complete summation machinery for representing (q)-hypergeometric products and mixed versions in difference rings defined over the our general field K (see above).

Joint work with Carsten Schneider

Trading field extensions and recurrence order through refined holonomic summation

Mark Round

RISC, Johannes Kepler University Linz, Austria

Solving definite nested multi-sums in terms of rational functions and generalizations of harmonic numbers through refined holonomic summation is introduced. In brief this is a generalization that allows inhomogeneous recurrences. Intermediate recurrences generated in solving a multi-sum are not solved in the refined holonomic approach. Furthermore, freedom afforded within the difference field theory, allows one to balance the number of field extensions required to write an inhomogeneous recurrence with the order of the recurrence. A formal exposition of these ideas is given with some illustrative examples and brief comparison to alternative methods.

Characterizations of $R\Pi\Sigma^*$-extensions

Carsten Schneider

RISC, Johannes Kepler University Linz, Austria

Recently, a new class of difference rings have been introduced. $R\Pi\Sigma^*$ enable one to represent expressions not only in terms of transcendental sums and products, like in Karr's difference field setting, but one can work also with products over primitive roots of unity. Since one can solve the parameterized telescoping problem in such rings, covering as special cases the summation paradigms of telescoping and creative telescoping, one obtains a rather flexible toolbox for symbolic summation. Inspired by Singer's Galois theory of difference equations we will work out several alternative characterizations of $R\Pi\Sigma^*$-extensions: adjoining naively sums and products leads to an $R\Pi\Sigma^*$-extension iff the obtained difference ring is simple iff the ring can be embedded into the ring of sequences or iff the ring can be given by the interlacing of $Pi\Sigma^*$-extensions. This will lead to several new applications in the context of symbolic summation. Some of them will be addressed in this talk.

Lifting techniques for Hermite-Padé approximants

Eric Schost

University of Waterloo

We consider the problem of Hermite-Padé approximation for power series with rational coefficients. To avoid issues of intermediate coefficient growth, a natural idea to handle these situations is to use p-adic techniques. We will discuss how some widely applicable ideas such as Newton iteration apply in this context, and to what extent improvements are possible for algebraic or D-finite power series.

An Algorithm For Computing Rational General Solutions of First-order Algebraic ODEs

N. Thieu Vo

RISC, Johannes Kepler University Linz, Austria

In this talk, we consider the class of first order algebraic differential equations (AODEs) and study their rational general solutions. We give a decision algorithm for computing a rational general solution, in which the constant appears rationally, of the whole class of first order AODEs. The method is based on the consideration of the ODE from the geometric point of view. In particular parametrizations of algebraic curves play an important role for a transformation of the given ODE to a quasi-linear differential equation.

This is joint work with Georg Grasegger and Franz Winkler.
