Dispersion and complexity of indefinite summation

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The notion of *dispersion* plays a crucial role in the development of modern algorithms for indefinite summation. First introduced by Abramov in his classical work [1] as the maximal integer distance between roots of the denominator of a reduced rational function, it has since been a key notion in several algorithmic developments.

Most of existing algorithms for rational and hypergeometric indefinite summation [2, 5, 3] exhibit dependency of the running time on the value of the dispersion which makes them unnecessarily slow for the cases of large dispersion and small output size (one exception is an algorithm from [4], see also [6] for details).

We analyze the relation between value of the dispersion and running time complexity of indefinite summation algorithms for different classes of summands, and show that the dependency of the running time on dispersion is non-essential (i.e. it is a feature of algorithms, not of the summation problems) in many cases. This leads to practical improvements of the algorithms for indefinite summation, based on ideas of direct indefinite summation [6]. Implementations of those improvements in Maple will be compared to standard Maple summation tools.

References

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