Lecture 16
Introduction to pattern recognition

1. What is pattern recognition
2. Stages of pattern recognition
3. Techniques for pattern recognition
4. Mathematical morphology
What is pattern recognition?

What is pattern recognition? the act of taking in raw image and taking an action based on the “category” of patterns shown in the data

• Patterns are elements or formation happened in repeatedly manner

• Example: sorting incoming Fish on a conveyor according to species using optical sensing”
How?

1. Set up a camera
2. Take some sample images
3. Process the images
4. Extract features
   - Length
   - Lightness
   - Width
   - Number and shape of fins
   - Position of the mouth, etc…
Pattern Classification System

• Preprocessing
  – Image processing
  – Segment (isolate) interested objects (fishes) from one another and from the background

• Feature Extraction
  – Reduce the data by measuring certain features

• Classification
  – Divide the feature space into decision region
  – Pattern representation and description
How to do Classification

- Example: use the length of the fish as a possible feature for discrimination
- The length is a poor feature alone!
- Select the lightness as a possible feature
Feature Selection

- The length is a poor feature alone!
- Select the lightness as a possible feature
Threshold decision

• Move decision boundary toward smaller values of lightness in order to minimize the cost (reduce the number of sea bass that are classified salmon!)

• Task of decision theory
Feature Vector

- Adopt the lightness and add the width of the fish to the feature vector $x^T = [x_1, x_2]$

- Straight line decision boundary
Is this a good decision boundary?
Decision Boundary Choice

- Our satisfaction is premature because the aim of designing a classifier is to correctly classify new (test) input.
- Better decision boundary.
Pattern Recognition Stages

• Sensing
  – Use of a transducer (camera or microphone)
  – PR system depends on the bandwidth, the resolution sensitivity
distortion of the transducer

• Segmentation and grouping
  – Patterns should be well separated and should not overlap

• Feature extraction
  – Discriminative features
  – Invariant features with respect to translation, rotation, and scale

• Classification
  – Use the feature vector provided by a feature extractor to assign
  the object to a category

• Post Processing
  – Exploit context-dependent information to improve performance
PR system Design Cycle

1. Data collection
2. Feature Choice
3. Model Choice
4. Training
5. Evaluation
6. Computational Complexity
Data Collection

• Data Collection
  – How do we know when we have collected an adequately large and representative set of examples for training and testing the system?

• Choice of Features
  – Depends on the characteristics of the problem domain
  – Simple to extract, invariant to irrelevant transformations, insensitive to noise

• Model Choice
  – Unsatisfied with the performance of our fish classifier and want to jump to another class of model

• Training
  – Use data to determine the classifier
  – (Many different procedures for training classifiers and choosing models)

• Evaluation
  – Measure the error rate (or performance)
  – Possibly switch from one set of features to another one

• Computational Complexity
  – What is the trade-off between computational ease and performance?
  – How does an algorithm scale as a function of the number of features, patterns, or categories?
Learning and Adaptation

• Supervised learning
  – A teacher provides a category label for each pattern in the training set

• Unsupervised learning
  – The system forms clusters or “natural groupings” of the unlabeled input patterns
Baysian Decision Theory

- Fundamental statistical approach
- Assumes relevant probabilities are known, compute the probability of the event observed, then make optimal decisions

- Bayes’ Theorem:  \( P(A | B) = \frac{P(B | A)P(A)}{P(B)} \)
- Example:
  Suppose at Laurier, 50% are girl students, 30% are science students, among science students, 20% are girl students. If one meet a girl student at Laurier, what is the probability that she is a science student.

  B – girl students, A – science students. Then

  \[
  P(A) = 30% \\
  P(B) = 50% \\
  P(B | A) = 20% \\
  P(A | B) = \frac{P(B | A)P(A)}{P(B)} = \frac{0.2 \times 0.3}{0.5} = 0.12 
  \]
Unsupervised Learning

• Often called clustering. The system is not given a set of labeled patterns for training. Instead the system establishes the classes itself based on the regularities of the patterns

• Clustering Separate Clouds
  – Methods work fine when clusters form well separated compact clouds
  – Less well when there are great differences in the number of samples in different clusters
Hierarchical Clustering

• Sometimes clusters are not disjoint, but may have subclusters, which in turn having sub-subclusters, etc.
• Consider partitioning $n$ samples into clusters
  • Start with $n$ cluster, each one containing exactly one sample
  • Then partition into $n-1$ clusters, then into $n-2$, etc.
Image pattern recognition

• To identify objects represented by images

• Techniques for image pattern recognition
  – Image pre-processing
  – Mathematical morphology
  – Segmentation
  – Object representation and description
  – Object recognition

• Investigate binary image. Binary image can be represented as a set of value 1 pixels. Set operations can be applied to the binary image

\[ C(x, y) = \begin{cases} 
1, & (x, y) \in A \\
0, & (x, y) \not\in A 
\end{cases} \]
What is Morphology

- A word from biology

- Mathematical morphology

  a tool for extracting image components that is useful in representation and description of region shape, boundaries, skeletons, convex hull

- Pre-, post-processing: morphological filtering, thinning, pruning
Basic concepts from set theory

Given sets $A$, $B$, subsets of rectangular set on X-Y plane

1. The complement of $A$:

$$A^c = \{ w \mid w \notin A \}$$

2. The union of $A$ and $B$:

$$A \cup B = \{ w \mid w \in A \text{ and } w \in B \}$$

3. The intersection of $A$ and $B$:

$$A \cap B = \{ w \mid w \in A \text{ or } w \in B \}$$

4. The difference of $A$ and $B$:

$$A - B = \{ w \mid w \in A \text{ and } w \notin B \}$$

5. The reflection of $A$:

$$\hat{A} = \{ w \mid w = -b, \text{ for } w \in A \}$$

6. The translate of $A$ by

$$(A)_z = \{ c \mid c = a + z, \text{ for } a \in A \} \quad z = (z_1, z_2)$$
Structure elements

- A small sets for subimages used to probe an image for properties of interest
- Examples

**Figure 9.2** First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.
How to probe

- Pad the image with 0’s to a rectangular image, large enough for element at the boundary
- Minimum pad the structure element
- Probe the image with the center of SE on each value 1 pixel of the image.

**Figure 9.3** (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.
Erosion

- Given set $A$ and $B$. The erosion of $A$ by $B$:

$$A \Theta B = \{z \mid (B)_z \subseteq A\}$$

**Figure 9.4** (a) Set $A$. (b) Square structuring element, $B$. (c) Erosion of $A$ by $B$, shown shaded. (d) Elongated structuring element. (e) Erosion of $A$ by $B$ using this element. The dotted border in (c) and (e) is the boundary of set $A$, shown only for reference.
**FIGURE 9.5** Using erosion to remove image components. (a) A $486 \times 486$ binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes $11 \times 11$, $15 \times 15$, and $45 \times 45$, respectively. The elements of the SEs were all 1s.
Dilation

- Given set $A$ and $B$. The dilation of $A$ by $B$:

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \}$$

**FIGURE 9.6**

(a) Set $A$.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of $A$ by $B$, shown shaded.
(d) Elongated structuring element.
(e) Dilation of $A$ using this element. The dotted border in (c) and (e) is the boundary of set $A$, shown only for reference.
Erosion

- Given set $A$ and $B$
- The erosion of $A$ by $B$: $A \ominus B = \{ z \mid (B)_z \subseteq A \}$
Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

**FIGURE 9.7**
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.
Duality

- Erosion and dilation are duals of each other

\[(A \Theta B)^c = A^c \oplus \hat{B}\]

\[(A \oplus B)^c = A^c \ominus \hat{B}\]
Opening

• The opening of $A$ by SE $B$

$$A \circ B = (A \Theta B) \oplus B$$

**FIGURE 9.8** (a) Structuring element $B$ “rolling” along the inner boundary of $A$ (the dot indicates the origin of $B$). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade $A$ in (a) for clarity.
Closing

The closing of $A$ by SE $B$

$$A \bullet B = (A \oplus B) \ominus B$$

**FIGURE 9.9** (a) Structuring element $B$ “rolling” on the outer boundary of set $A$. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade $A$ in (a) for clarity.
Properties of opening and closing

\[(A \bullet B)^c = A^c \circ \hat{B}\]
\[(A \circ B)^c = A^c \bullet \hat{B}\]

\[A \circ B \subseteq A\]

if  \(C \subseteq D\), then \(C \circ B \subseteq D \circ B\)

\[(A \circ B) \circ B = A \circ B\]

\[A \bullet B \subseteq A\]

if  \(C \subseteq D\), then \(C \bullet B \subseteq D \bullet B\)

\[(A \bullet B) \bullet B = A \bullet B\]
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.
Construct filters by morphological operation

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)
Hit or miss transformation

- A basic tool for shape detection

\[ A \ast B = (A \Theta B) \cap [A^c \Theta (W - D)] \]

**FIGURE 9.12**
(a) Set \( A \).
(b) A window, \( W \), and the local background of \( D \) with respect to \( W, (W - D) \).
(c) Complement of \( A \).
(d) Erosion of \( A \) by \( D \).
(e) Erosion of \( A^c \) by \( (W - D) \).
(f) Intersection of (d) and (e), showing the location of the origin of \( D \), as desired. The dots indicate the origins of \( C, D \), and \( E \).
Basic morphological algorithm

- Extracting boundaries, connected components, the convex hull, and skeleton of a region
- Boundary extraction: $\beta(A) = A - (A \Theta B)$
FIGURE 9.14
(a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).
Hole filling

- Let $X_0$ be all 0’s. Continue the following computing

$$X_k = (X_{k-1} \oplus B) \cap A^c, \ k = 1, 2, 3, \ldots$$

until $X_k = X_{k-1}$
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.
Extraction of connected components

- Let $X_0$ be all 0’s. Continue the following computing

$$X_k = (X_{k-1} \oplus B) \cap A, \ k = 1, 2, 3, \ldots$$

until $X_k = X_{k-1}$
FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a $5 \times 5$ structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geräte GmbH, Diepholz, Germany, www.ntbxray.com.)
Convex hull

- Let $X_0$ be all 0’s. Continue the following computing

$$X_0^i = A, i = 1, 2, 3, 4$$

$$X_k^i = (X_{k-1}^i \ast B^i) \cap A, i = 1, 2, 3, 4, k = 1, 2, 3...$$

until $X_k^i = X_{k-1}^i$, then let $D^i = X_k^i$

$$C(A) = \bigcup_{i=1}^{4} D^i$$
FIGURE 9.19
(a) Structuring elements. (b) Set $A$. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.
Thinning and Thickening

• The thinning of A by SE B

\[ A \otimes B = A - (A \ast B) = A \cap (A \ast B)^c \]

• Thinning by a sequence of SE

\[ \{B\} = \{B^1, B^2, B^3, ..., B^n\} \]

\[ A \otimes \{B\} = (\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n \]
FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set $A$. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to $m$-connectivity.
Thickening

• The thickening of A by SE B

\[ A \square B = A \cup (A \ast B) \]

• Thickening by a sequence of SE

\[ \{B\} = \{B^1, B^2, B^3, ..., B^n\} \]

\[ A \square \{B\} = (((A \square B^1) \square B^2)...) \square B^n \]

**FIGURE 9.22** (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.
Skeletons

\[ S(A) = \bigcup_{k=0}^{K} S_k(A) \]

\[ S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B \]

\[ (A \Theta kB) = (((A \Theta B) \Theta B) \Theta ...) \Theta B) \]

\[ K = \max \{ k \mid (A \Theta kB) \neq \emptyset \} \]

**FIGURE 9.23**
(a) Set \( A \).
(b) Various positions of maximum disks with centers on the skeleton of \( A \).
(c) Another maximum disk on a different segment of the skeleton of \( A \).
(d) Complete skeleton.
**FIGURE 9.24**
Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.
Pruning

• Pruning is done by the following four steps

\[ X_1 = A \otimes \{B\} \]
\[ X_2 = \bigcup_{k=1}^{8} (X_1 \ast B^k) \]
\[ X_3 = (X_2 \oplus H) \cap A \]
\[ X_4 = X_1 \cup X_3 \]
Morphological Reconstruction

• Geodesic dilation
  \[ D_G^{(1)}(F) = (F \oplus B) \cap G \]
  \[ D_G^{(n)}(F) = D_G^{(1)}(F)[D_G^{(n-1)}(F)] \]

• Morphological reconstruction by dilation
  \[ R_G^D(F) = D_G^{(k)}(F), \text{where } D_G^{(k)}(F) = D_G^{(k-1)}(F) \]

• Geodesic erosion
  \[ E_G^{(1)}(F) = (F \ominus B) \cup G \]
  \[ E_G^{(n)}(F) = E_G^{(1)}(F)[E_G^{(n-1)}(F)] \]

• Morphological reconstruction by erosion
  \[ R_G^E(F) = E_G^{(k)}(F), \text{where } E_G^{(k)}(F) = E_G^{(k-1)}(F) \]
FIGURE 9.26
Illustration of geodesic dilation.
FIGURE 9.27
Illustration of geodesic erosion.
FIGURE 9.28
Illustration of morphological reconstruction by dilation. $F$, $G$, $B$ and $D_G^{(1)}(F)$ are from Fig. 9.26.
Simple applications

Segmentation of nontrivial images is one of the most difficult problems in computer vision. Segmentation accuracy is crucial for effective and efficient analysis procedures. For this reason, it is important to improve the probability of correct segmentation. Segmentation is a critical part of computerized analysis procedures. For this reason, it is important to improve the probability of correct segmentation. For this reason, it is important to improve the probability of correct segmentation.

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tallest character is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.
**FIGURE 9.30**
Illustration of hole filling on a simple image.
FIGURE 9.31
(a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).
FIGURE 9.32
Border clearing.
(a) Marker image.
(b) Image with no objects touching the border. The original image is Fig. 9.29(a).
FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×’s indicate “don’t care” values.
<table>
<thead>
<tr>
<th>Operation</th>
<th>Equation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation</td>
<td>((D)z = {w \mid w - b \perp z, \text{ for } b \neq R})</td>
<td>Translates the origin of (B) to point (z).</td>
</tr>
<tr>
<td>Reflection</td>
<td>(\hat{B} = {w \mid w = -b, \text{ for } b \in B})</td>
<td>Reflects all elements of (B) about the origin of this set.</td>
</tr>
<tr>
<td>Complement</td>
<td>(A^c = {w \mid w \not\in A})</td>
<td>Set of points not in (A).</td>
</tr>
<tr>
<td>Difference</td>
<td>(A - B = {w \mid w \in A, w \not\in B} = A \cap B^c)</td>
<td>Set of points that belong to (A) but not to (B).</td>
</tr>
<tr>
<td>Dilation</td>
<td>(A \oplus B = {z \mid (\hat{B}_z) \cap A \neq \emptyset})</td>
<td>“Expands” the boundary of (A). (I)</td>
</tr>
<tr>
<td>Erosion</td>
<td>(A \ominus B = {z \mid (B)_z \subseteq A})</td>
<td>“Contracts” the boundary of (A). (I)</td>
</tr>
<tr>
<td>Opening</td>
<td>(A \circ B = (A \oplus B) \ominus B)</td>
<td>Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
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<th>Equation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closing</td>
<td>( A \cdot B = (A \oplus B) \ominus B )</td>
<td>Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)</td>
</tr>
<tr>
<td>Hit-or-miss transform</td>
<td>( A \ominus B = (A \ominus B_1) \cap (A' \ominus B) )</td>
<td>The set of points (coordinates) at which, simultaneously, ( B_1 ) found a match (&quot;hit&quot;) in ( A ) and ( B_2 ) found a match in ( A' ).</td>
</tr>
<tr>
<td>Boundary extraction</td>
<td>( \beta(A) = A - (A \ominus B) )</td>
<td>Set of points on the boundary of set ( A ). (I)</td>
</tr>
<tr>
<td>Hole filling</td>
<td>( X_k = (X_{k-1} \ominus B) \cap A ); ( k = 1, 2, 3, \ldots )</td>
<td>Fills lists in ( A ); ( X_0 = ) array of 1s with a 1 in each hole. (II)</td>
</tr>
<tr>
<td>Connected components</td>
<td>( X_k = (X_{k-1} \ominus B) \cap A ); ( k = 1, 2, 3, \ldots )</td>
<td>Finds connected components in ( A ); ( X_0 = ) array of 1s with a 1 in each connected component. (I)</td>
</tr>
<tr>
<td>Convex hull</td>
<td>( X_{k+1}^k = (X_k \ominus B) \cup A ); ( k = 1, 2, 3, \ldots )</td>
<td>Finds the convex hull ( C(A) ) of set ( A ), where &quot;conv&quot; indicates convergence in the sense that ( X_k \rightarrow X_{k+1}^k ). (III)</td>
</tr>
<tr>
<td>Thinning</td>
<td>( A \ominus B = A - (A \ominus B)^c )</td>
<td>Thins set ( A ). The first two equations give the basic definition of thinning. The last four equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)</td>
</tr>
<tr>
<td>Thickening</td>
<td>( A \ominus B = A \cup (A \ominus B)^c )</td>
<td>Thickens set ( A ). (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.</td>
</tr>
<tr>
<td>Skeletons</td>
<td>( S(A) = \bigcup_{k=0}^{K} S_k(A) )</td>
<td>Finds the skeleton ( S(A) ) of set ( A ). The last equation indicates that ( A ) can be reconstructed from its skeleton subsets ( S_k(A) ). (In all three equations, ( K ) is the value of the iterative step after which the set ( A ) erodes to the empty set. The notation ( (A \ominus B) ) denotes the ( k )th iteration of successive erosions of ( A ) by ( B ). (I)</td>
</tr>
<tr>
<td>Reconstruction of ( A ):</td>
<td>( A = \bigcup_{k=0}^{K} S_k(A) \ominus kB )</td>
<td></td>
</tr>
<tr>
<td>Hole filling</td>
<td>( H = [R^0_F(F)]^c )</td>
<td>( H ) is equal to the input image ( I ), but with all holes filled. See Eq. (9.5-28) for the definition of the marker image ( F ).</td>
</tr>
<tr>
<td>Border clearing</td>
<td>( X = I - R^0_F(F) )</td>
<td>( X ) is equal to the input image ( I ), but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image ( F ).</td>
</tr>
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Gray-Scale Morphology

- **Structure element**
- **Dilation** \[ (f \oplus b)(x, y) = \max_{(s,t)\in b} \{ f(x-s, y-t) \} \]
- **Erosion** \[ (f \ominus b)(x, y) = \min_{(s,t)\in b} \{ f(x+s, y+t) \} \]
- **Opening** \[ f \circ b = (f \Theta b) \oplus b \]
- **Closing** \[ f \bullet b = (f \oplus b) \Theta b \]
FIGURE 9.35  (a) A gray-scale X-ray image of size $448 \times 425$ pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)
FIGURE 9.37  (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.
FIGURE 9.38
(a) 566 \times 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope.
(b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)
FIGURE 9.39
(a) $512 \times 512$ image of a head CT scan.
(b) Dilation.
(c) Erosion.
(d) Morphological gradient, computed as the difference between (b) and (c).
(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)
FIGURE 9.40 Using the top-hat transformation for shading correction. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.
FIGURE 9.41  (a) $531 \times 675$ image of wood dowels. (b) Smoothed image. (c) (f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)
FIGURE 9.43
Textural segmentation.
(a) A $600 \times 600$ image consisting of two types of blobs.
(b) Image with small blobs removed by closing (a).
(c) Image with light patches between large blobs removed by opening (b).
(d) Original image with boundary between the two regions in (c) superimposed.
The boundary was obtained using a morphological gradient operation.
FIGURE 9.44 (a) Original image of size $1134 \times 1360$ pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)