Lecture 16
Introduction to pattern recognition

1. What is pattern recognition
2. Stages of pattern recognition
3. Techniques for pattern recognition

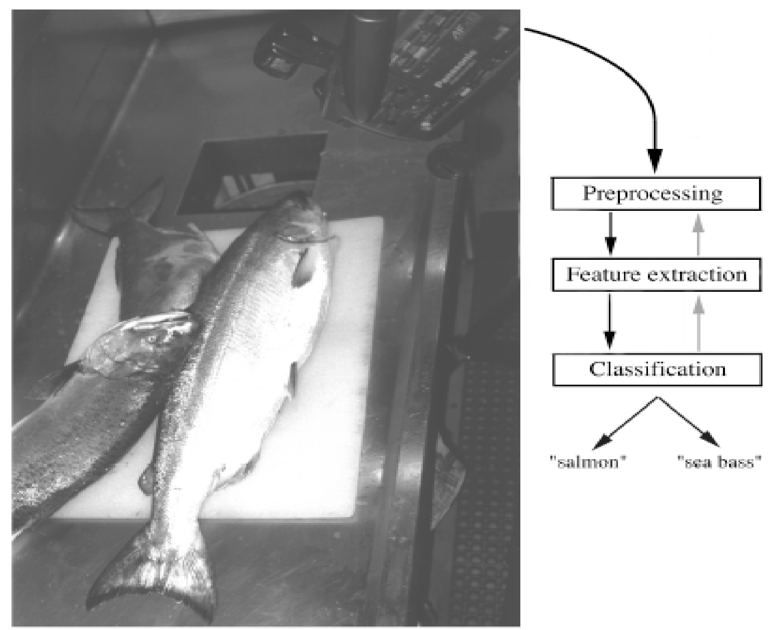
4. Mathematical morphology

What is pattern recognition?

What is pattern recognition? the act of taking in raw image and taking an action based on the “category” of patterns shown in the data

- Patterns are elements or formation happened in repeatedly manner

- Example: sorting incoming Fish on a conveyor according to species using optical sensing”



How?

1. Set up a camera
2. Take some sample images
3. Process the images
4. Extract features
   - Length
   - Lightness
   - Width
   - Number and shape of fins
   - Position of the mouth, etc…

Pattern Classification System

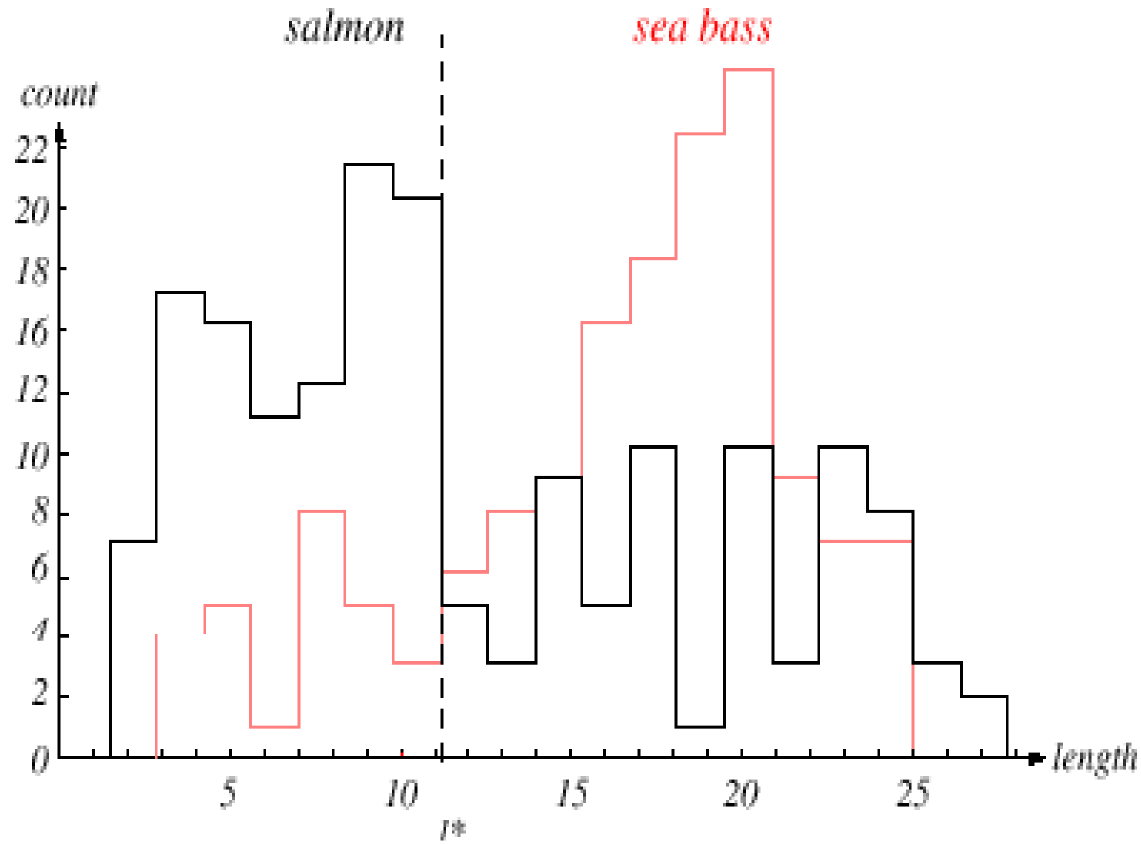
- Preprocessing
  - Image processing
  - Segment (isolate) interested objects (fishes) from one another and from the background

- Feature Extraction
  - Reduce the data by measuring certain features

- Classification
  - Divide the feature space into decision region
  - Pattern representation and description

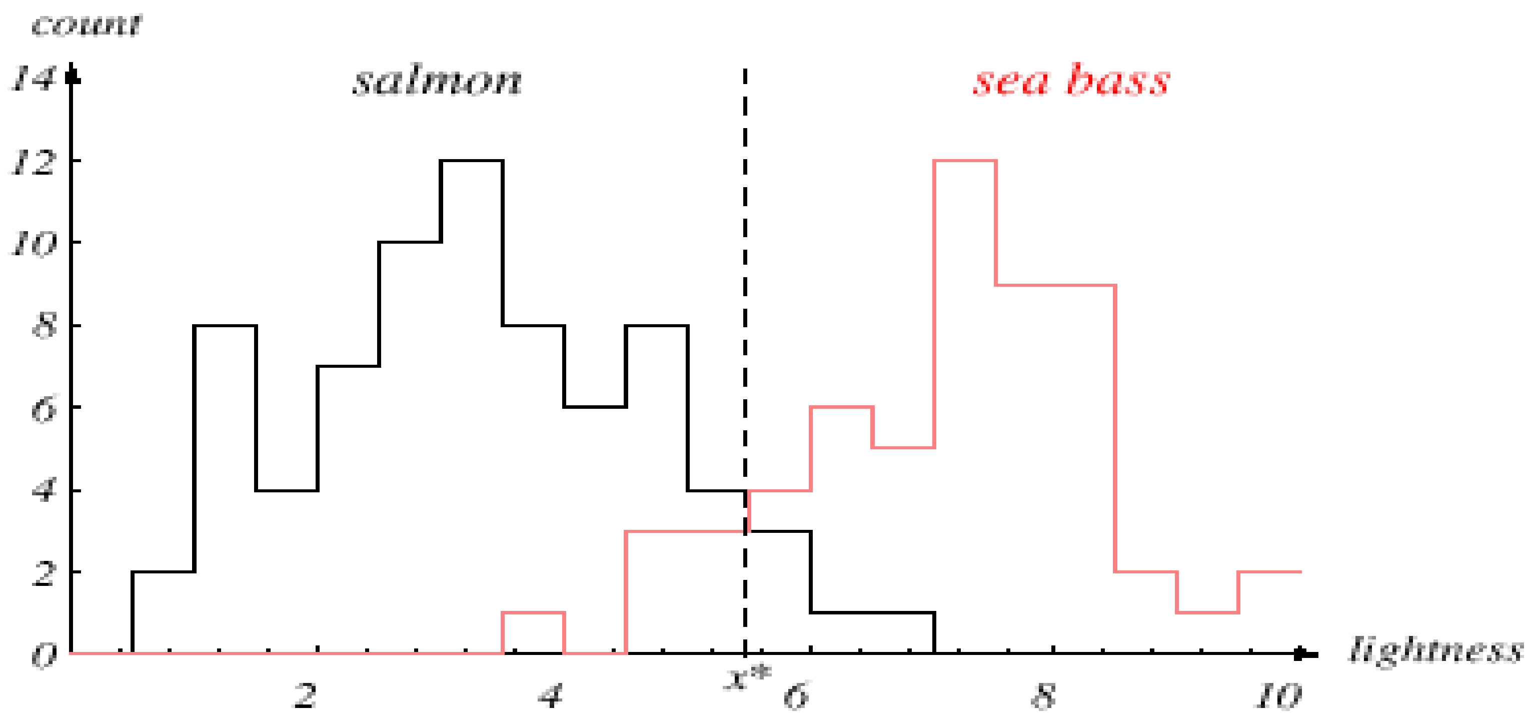
How to do Classification

- Example: use the length of the fish as a possible feature for discrimination
- The length is a poor feature alone!
- Select the lightness as a possible feature



Feature Selection

- The length is a poor feature alone!
- Select the lightness as a possible feature



Threshold decision

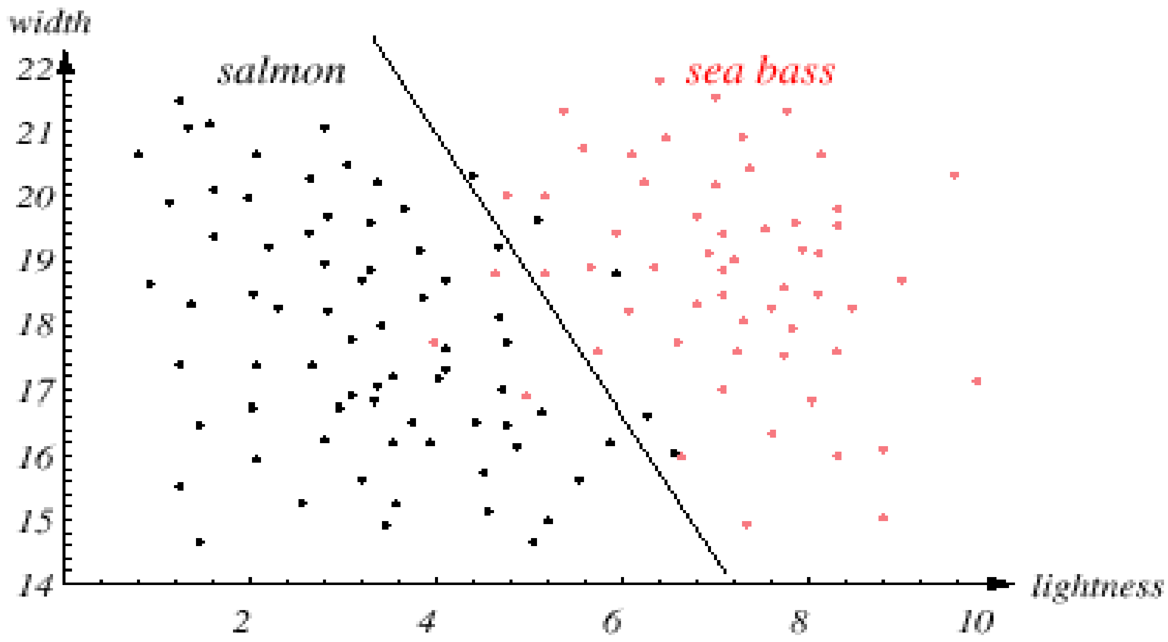
- Move decision boundary toward smaller values of lightness in order to minimize the cost (reduce the number of sea bass that are classified salmon!)

- Task of decision theory

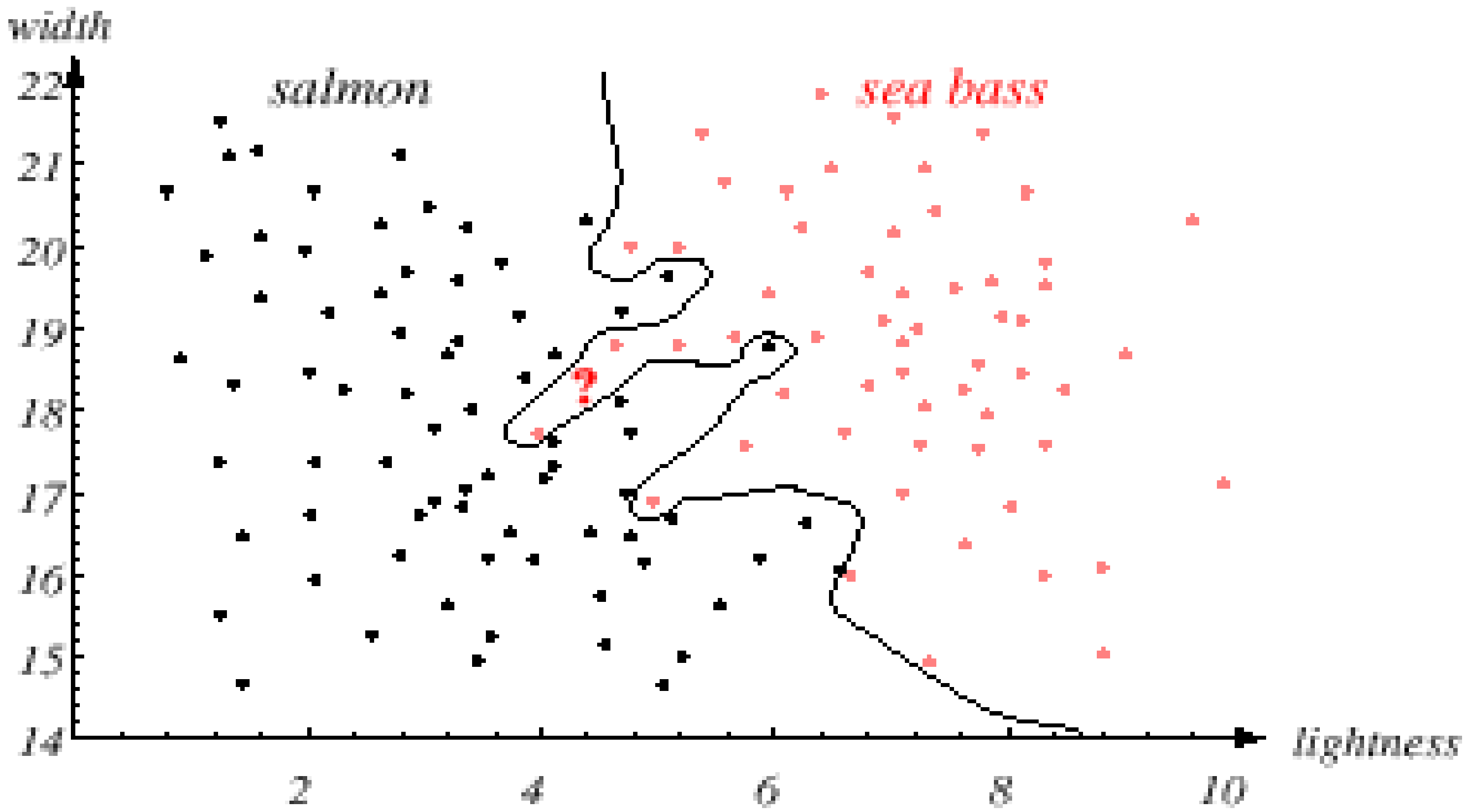
Feature Vector

- Adopt the lightness and add the width of the fish to the feature vector $x^T = [x_1, x_2]$

- Straight line decision boundary

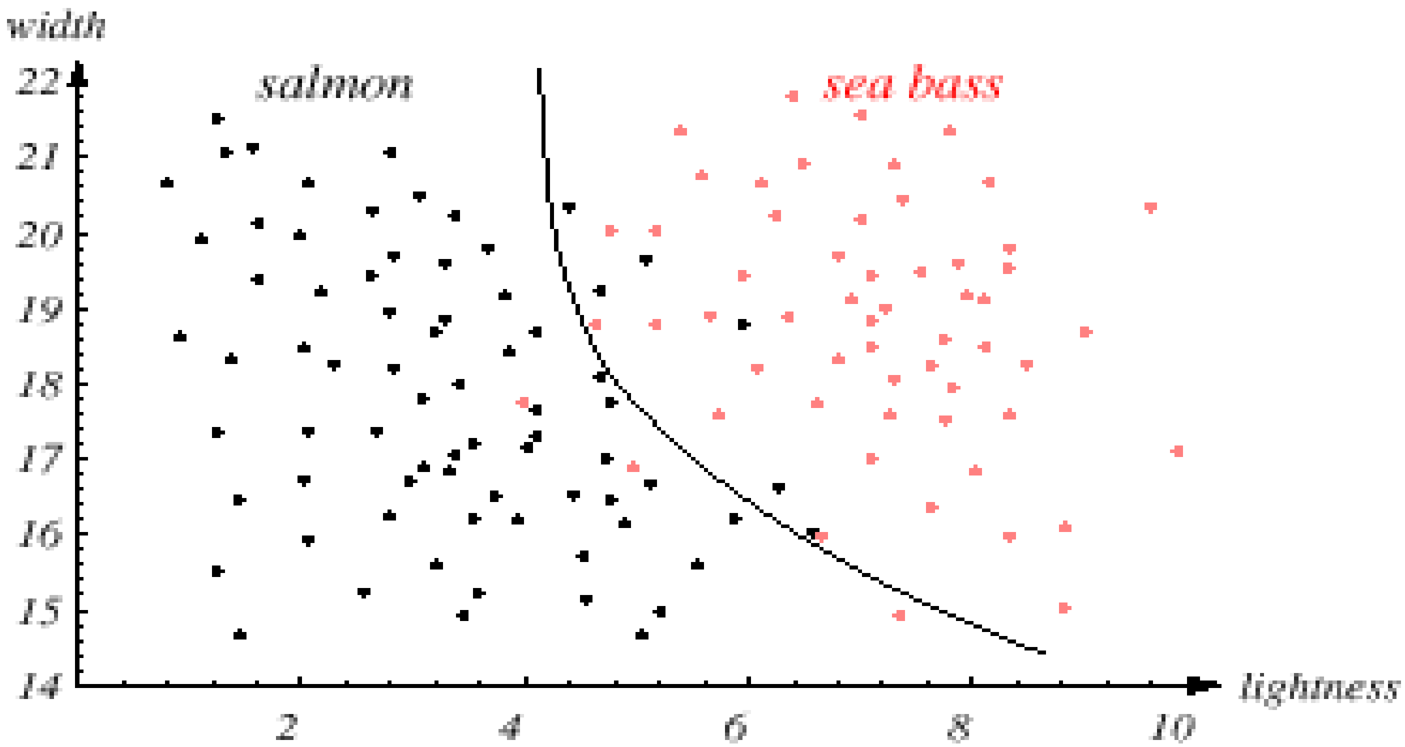


Is this a good decision boundary?



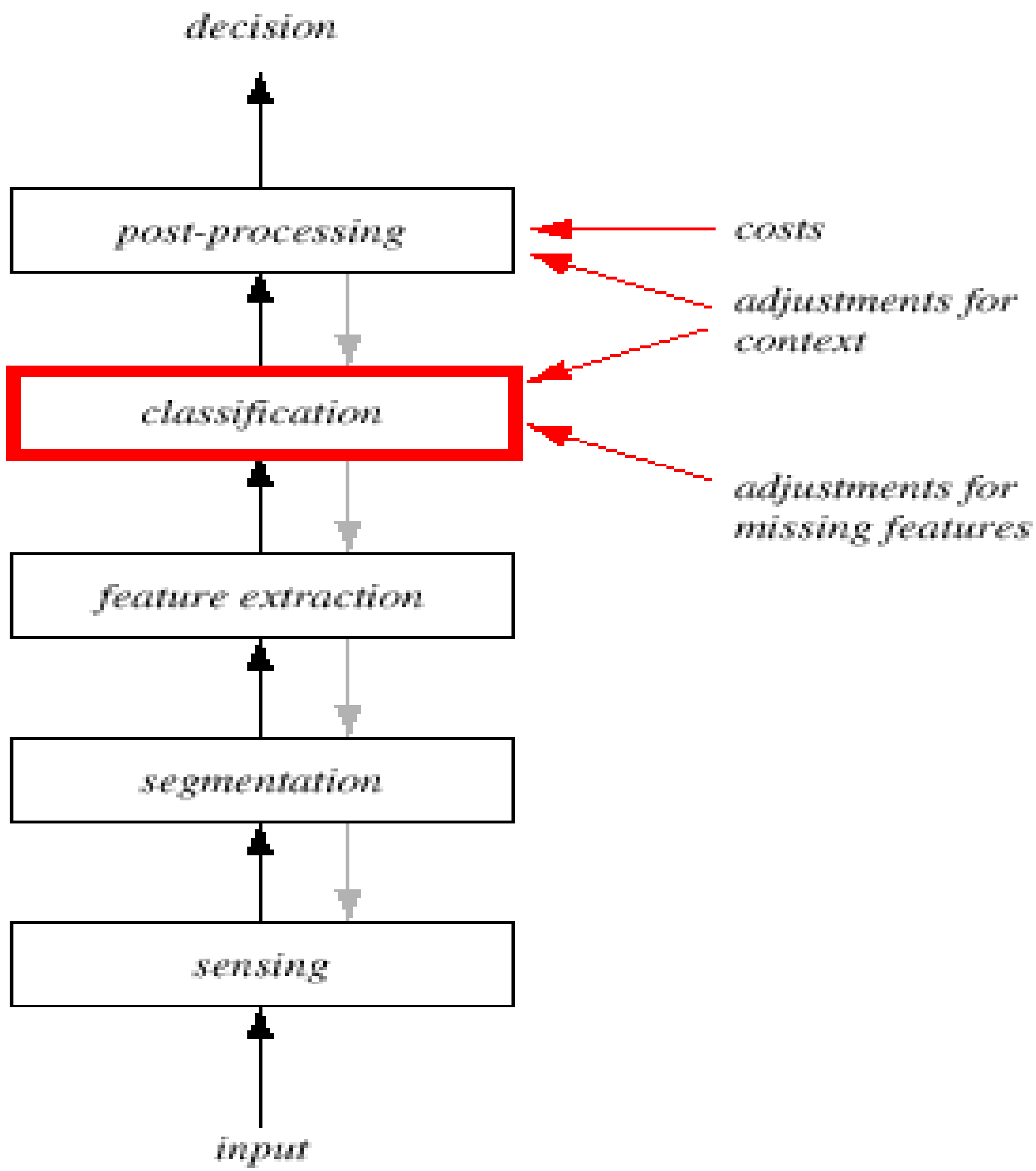
Decision Boundary Choice

- Our satisfaction is premature because the aim of designing a classifier is to correctly classify new (test) input
- Better decision boundary



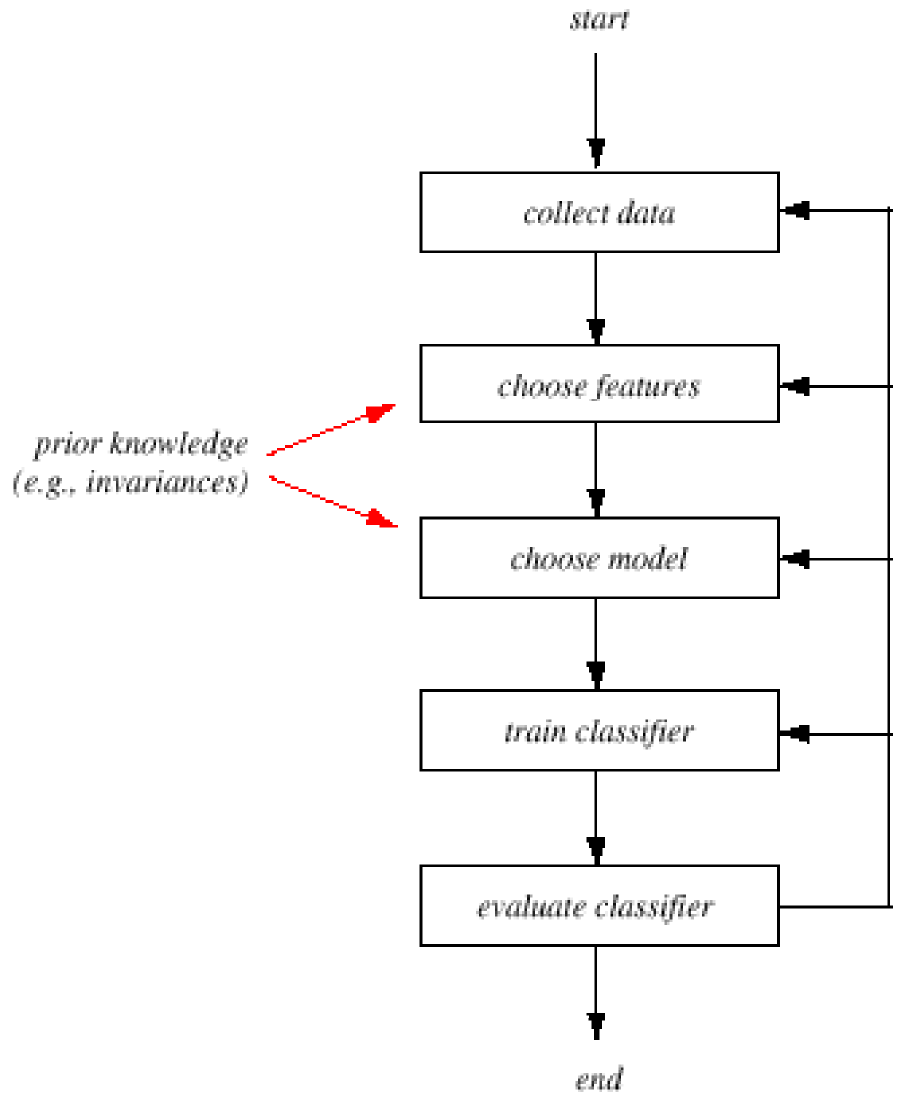
Pattern Recognition Stages

- Sensing
  - Use of a transducer (camera or microphone)
  - PR system depends on the bandwidth, the resolution sensitivity distortion of the transducer
- Segmentation and grouping
  - Patterns should be well separated and should not overlap
- Feature extraction
  - Discriminative features
  - Invariant features with respect to translation, rotation, and scale
- Classification
  - Use the feature vector provided by a feature extractor to assign the object to a category
- Post Processing
  - Exploit context-dependent information to improve performance


decision
post-processing
costs
adjustments for context
classification
adjustments for missing features
feature extraction
segmentation
sensing
input

PR system Design Cycle

1. Data collection
2. Feature Choice
3. Model Choice
4. Training
5. Evaluation
6. Computational Complexity



Data Collection

- Data Collection
  - How do we know when we have collected an adequately large and representative set of examples for training and testing the system?
- Choice of Features
  - Depends on the characteristics of the problem domain
  - Simple to extract, invariant to irrelevant transformations, insensitive to noise
- Model Choice
  - Unsatisfied with the performance of our fish classifier and want to jump to another class of model
- Training
  - Use data to determine the classifier
  - (Many different procedures for training classifiers and choosing models)
- Evaluation
  - Measure the error rate (or performance)
  - Possibly switch from one set of features to another one
- Computational Complexity
  - What is the trade-off between computational ease and performance?
  - How does an algorithm scale as a function of the number of features, patterns, or categories?

Learning and Adaptation

- Supervised learning
  - A teacher provides a category label for each pattern in the training set
- Unsupervised learning
  - The system forms clusters or “natural groupings” of the unlabeled input patterns

Baysian Decision Theory

- Fundamental statistical approach
- Assumes relevant probabilities are known, compute the probability of the event observed, then make optimal decisions
- Bayes' Theorem: $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)}$
- Example:

Suppose at Laurier, 50% are girl students, 30% are science students, among science students, 20% are girl students. If one meet a girl student at Laurier, what is the probability that she is a science student.

B – girl students, A – science students. Then

$$P(A) = 30\%$$

$$P(B) = 50\%$$

$$P(B \mid A) = 20\%$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} = \frac{0.2 \times 0.3}{0.5} = \frac{0.06}{0.5} = 0.12$$

Unsupervised Learning

- Often called clustering. The system is not given a set of labeled patterns for training. Instead the system establishes the classes itself based on the regularities of the patterns
- Clustering Separate Clouds
  - Methods work fine when clusters form well separated compact clouds
  - Less well when there are great differences in the number of samples in different clusters

Hierarchical Clustering

- Sometimes clusters are not disjoint, but may have subclusters, which in turn having sub-subclusters, etc.
- Consider partitioning *n* samples into clusters
  - Start with *n* cluster, each one containing exactly one sample
  - Then partition into *n-1* clusters, then into *n-2*, etc.

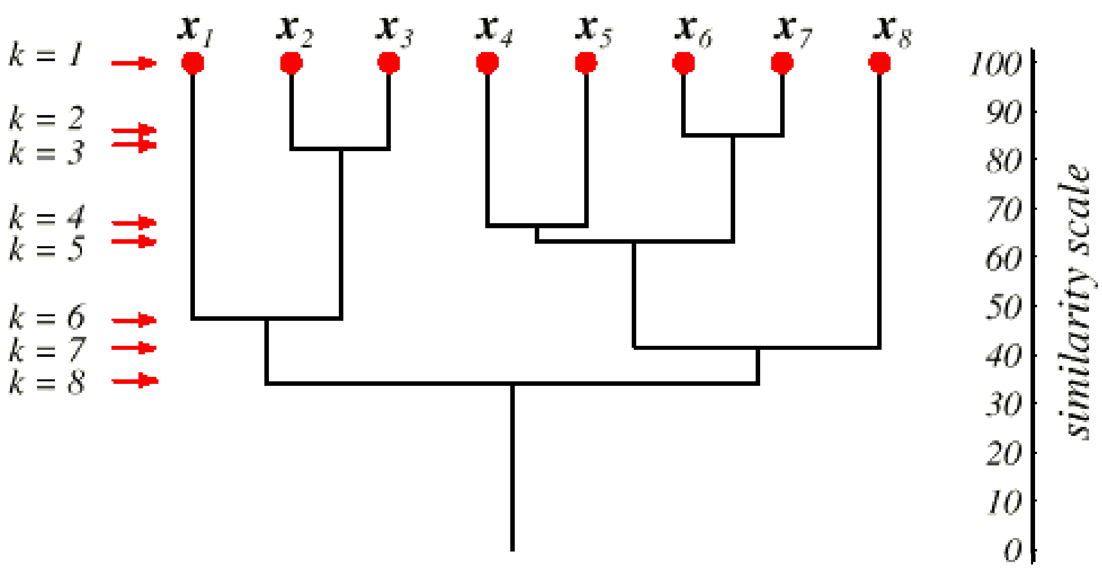


Image pattern recognition

- To identify objects represented by images

- Techniques for image pattern recognition
  - Image pre-processing
  - Mathematical morphology
  - Segmentation
  - Object representation and description
  - Object recognition

- Investigate binary image. Binary image can be represented as a set of value 1 pixels. Set operations can be applied to the binary image

$$C(x,y)=\begin{cases}1, & (x,y)\in A\\ 0, & (x,y)\notin A\end{cases}$$

What is Morphology

- A word from biology

- Mathematical morphology

  a tool for extracting image components that is useful in representation and description of region shape, boundaries, skeletons, convex hull

- Pre-, post-processing: morphological filtering, thinning, pruning

Basic concepts from set theory

Given sets A, B, subsets of rectangular set on X-Y plane

1. The complement of A:

$$A^c = \{w \mid w \notin A\}$$

2. The union of A and B:

$$A \cup B = \{w \mid w \in A \; and \;\; w \in B\}$$

3. The intersection of A and B:

$$A \cap B = \{w \mid w \in A \; or \; w \in B\}$$

4. The difference of A and B:

$$A - B = \{w \mid w \in A \; and \;\; w \notin B\}$$

5. The reflection of A:

$$\hat{A} = \{w \mid w = -b, \;\; for \; w \in A\}$$

6. The translate of A by

$$(A)_z = \{c \mid c = a + z, \;\; for \; a \in A\} \quad z = (z_1, z_2):$$

Structure elements

- A small sets for subimages used to probe an image for properties of interest
- Examples

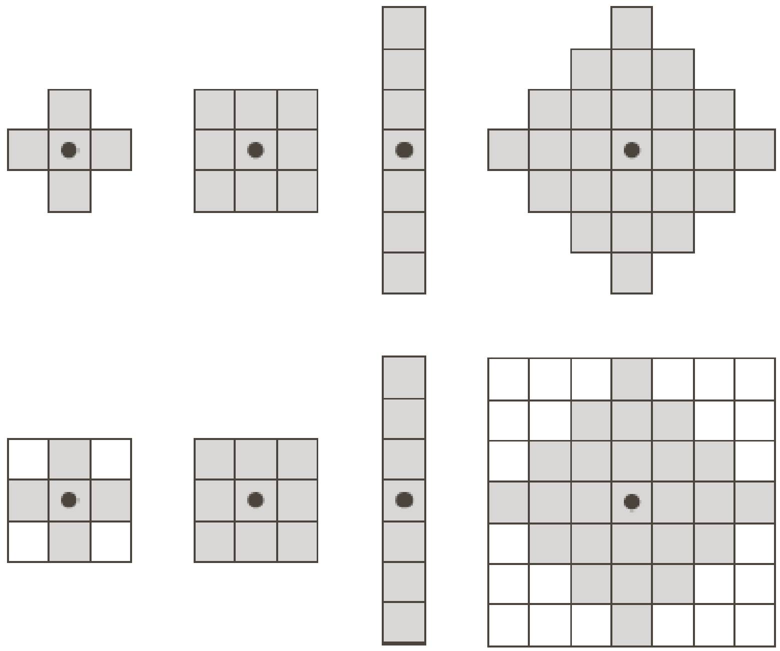
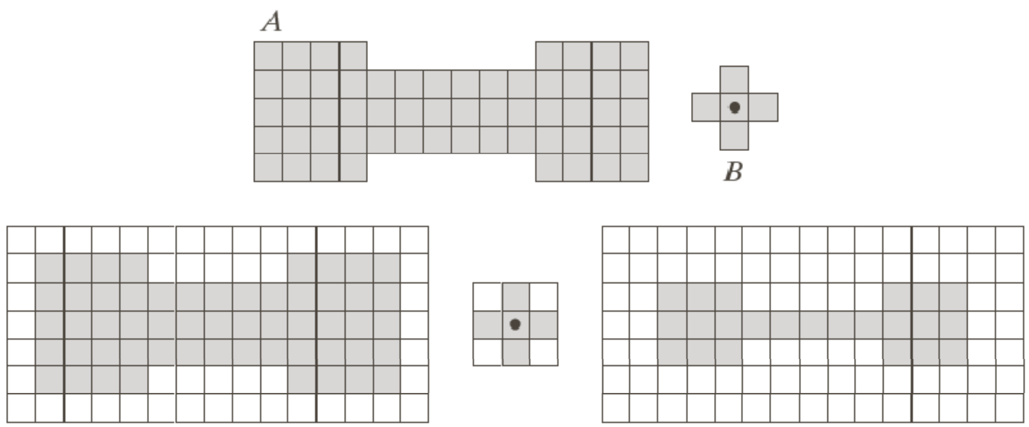


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

How to probe

- Pad the image with 0’s to a rectangular image, large enough for element at the boundary
- Minimum pad the structure element
- Probe the image with the center of SE on each value 1 pixel of the image.



a b
c d e

FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

Erosion

- Given set A and B. The erosion of A by B:

$$A \Theta B = \{z \mid (B)_z \subseteq A\}$$

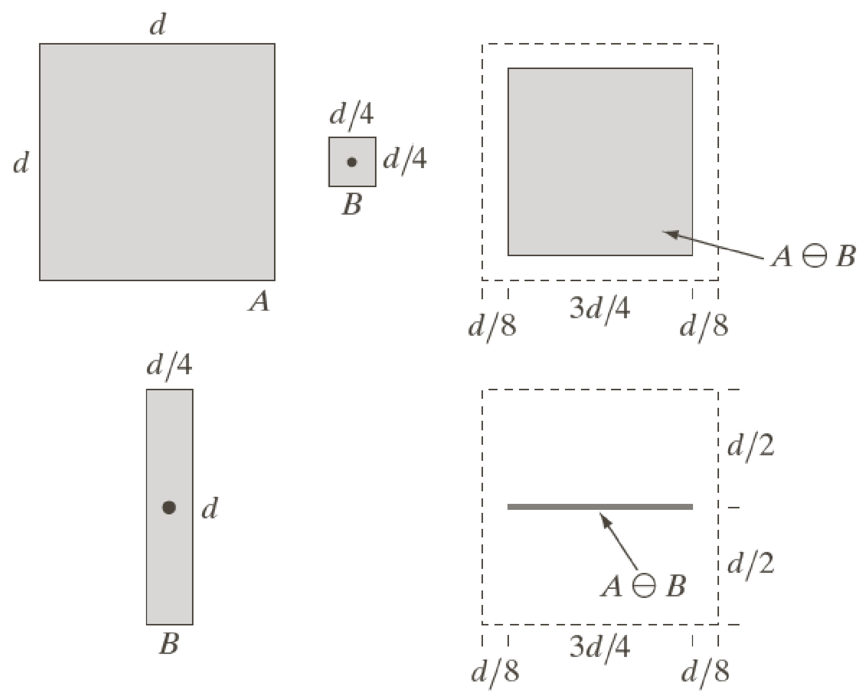
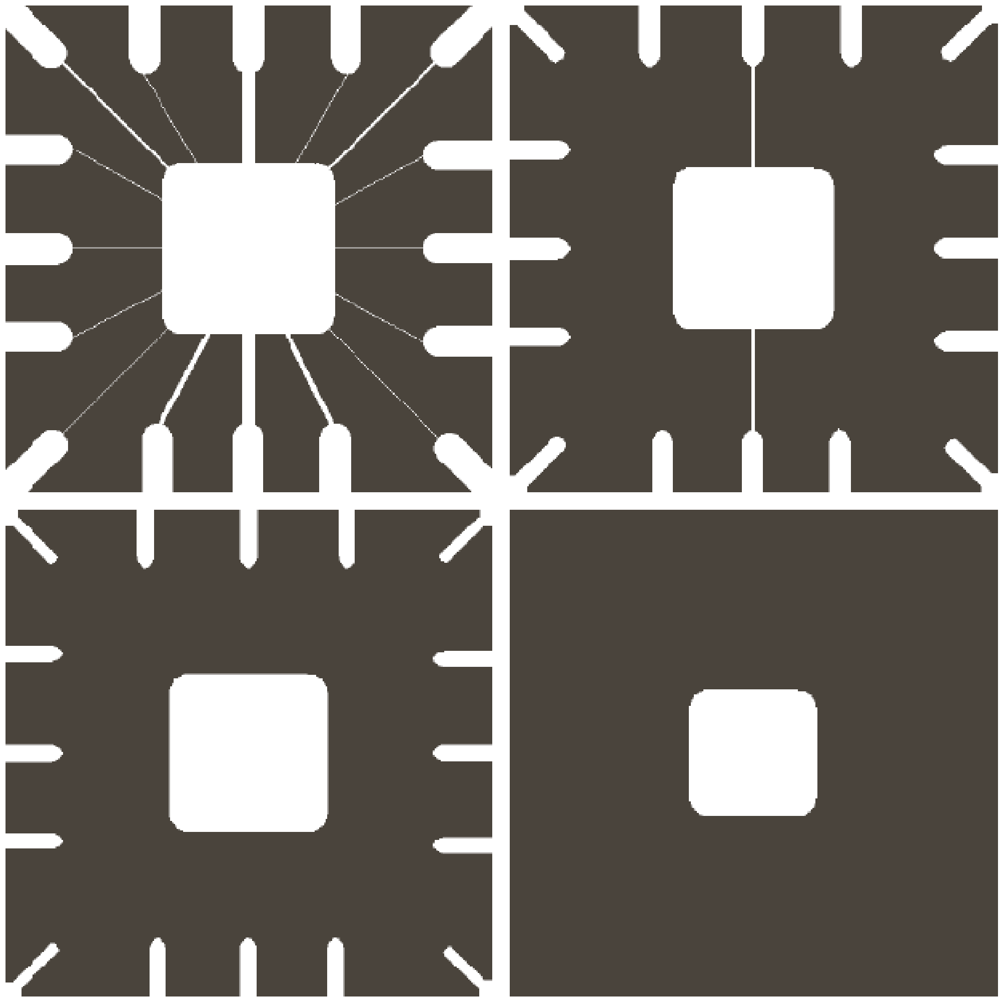


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.



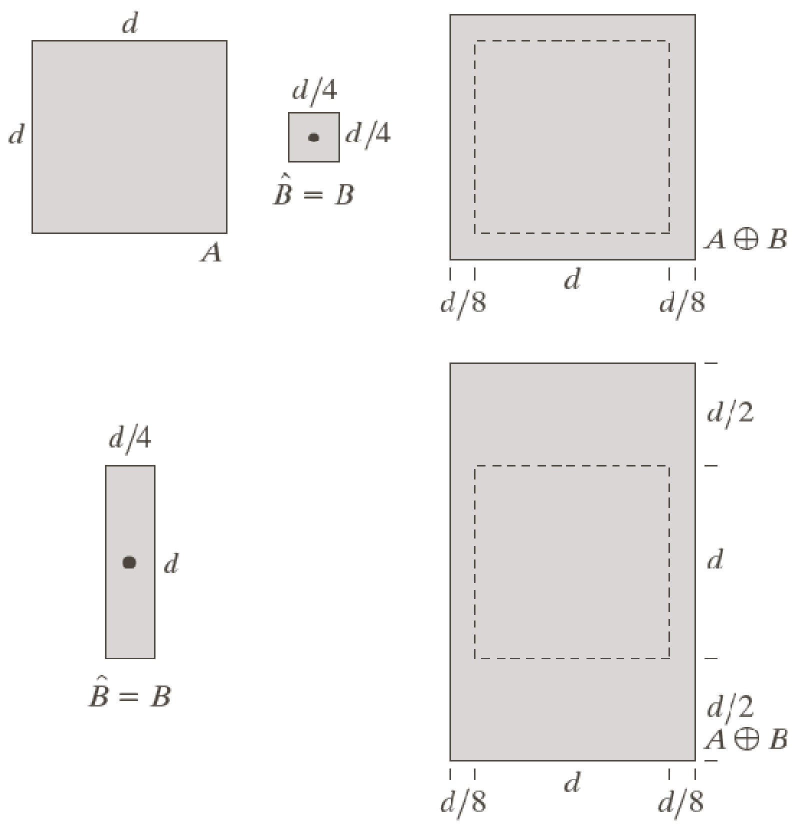
a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Dilation

- Given set A and B. The dilation of A by B:

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

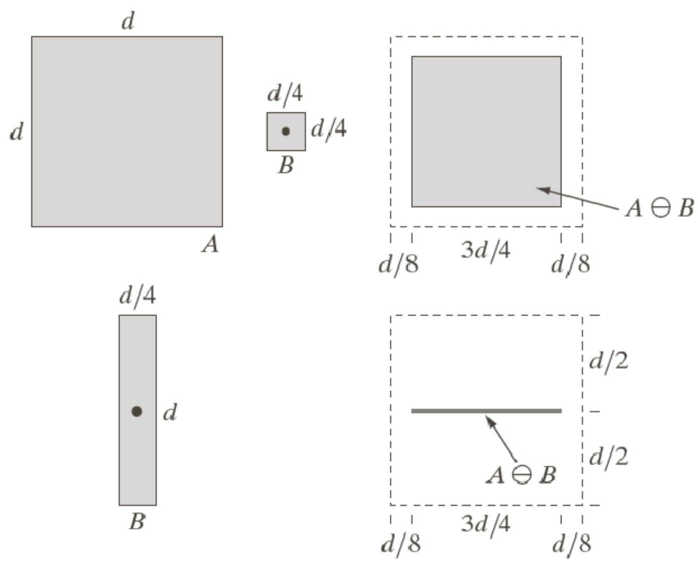


a b c
d e

FIGURE 9.6
(a) Set A.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element. (e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference

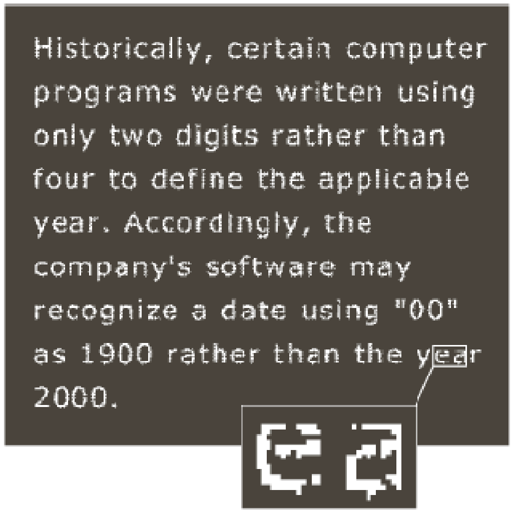
Erosion

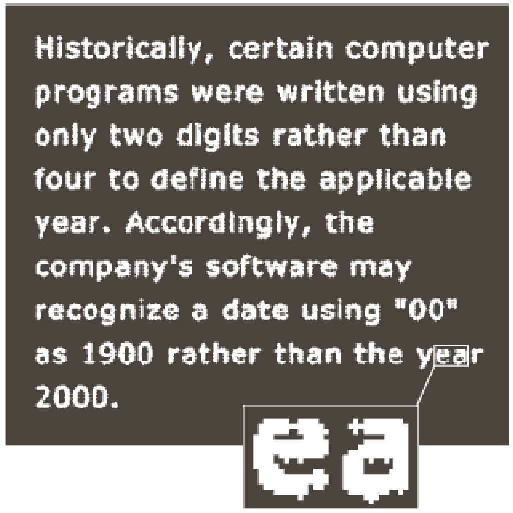
- Given set A and B
- The erosion of A by B: $A \Theta B = \{z \mid (B)_z \subseteq A\}$



a b c
d e

FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.





0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Duality

- Erosion and dilation are duals of each other

$$(A \Theta B)^c = A^c \oplus \hat{B}$$

$$(A \oplus B)^c = A^c \Theta \hat{B}$$

Opening

- The opening of A by SE B

$$A \circ B = (A \ominus B) \oplus B$$

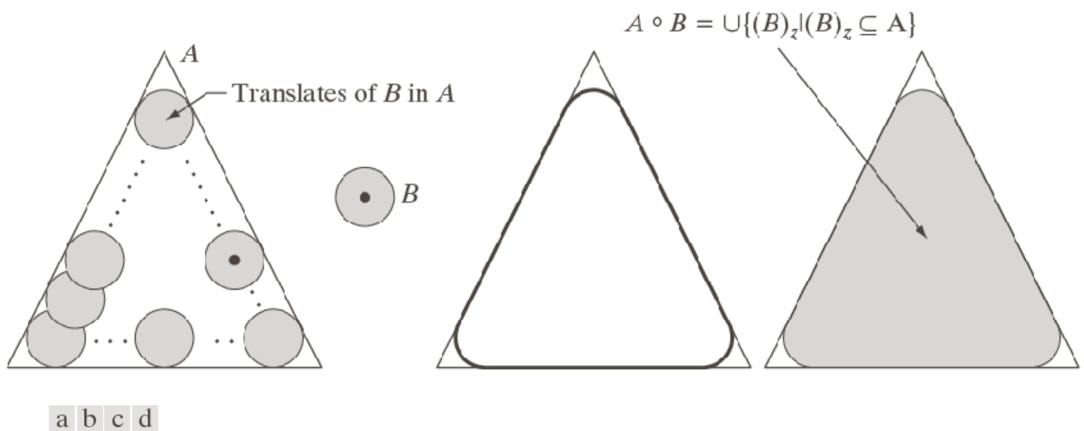


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.

Closing

The closing of A by SE B

$$A \bullet B = (A \oplus B) \ominus B$$

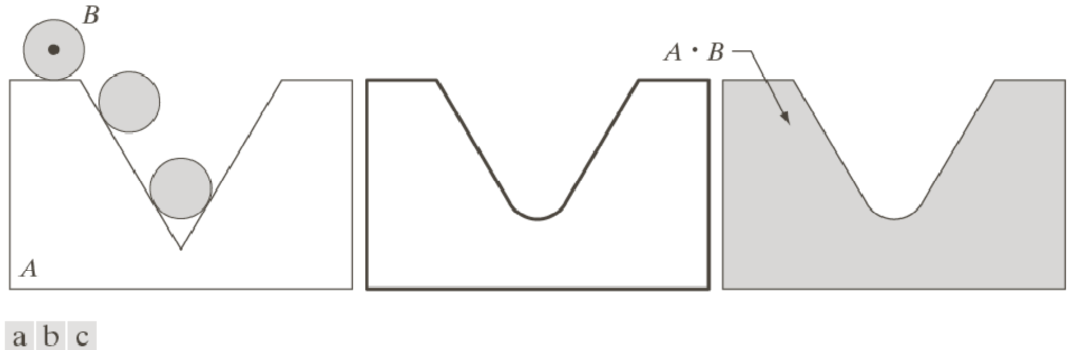


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

Properties of opening and closing

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = A^c \bullet \hat{B}$$

$$A \circ B \subseteq A$$

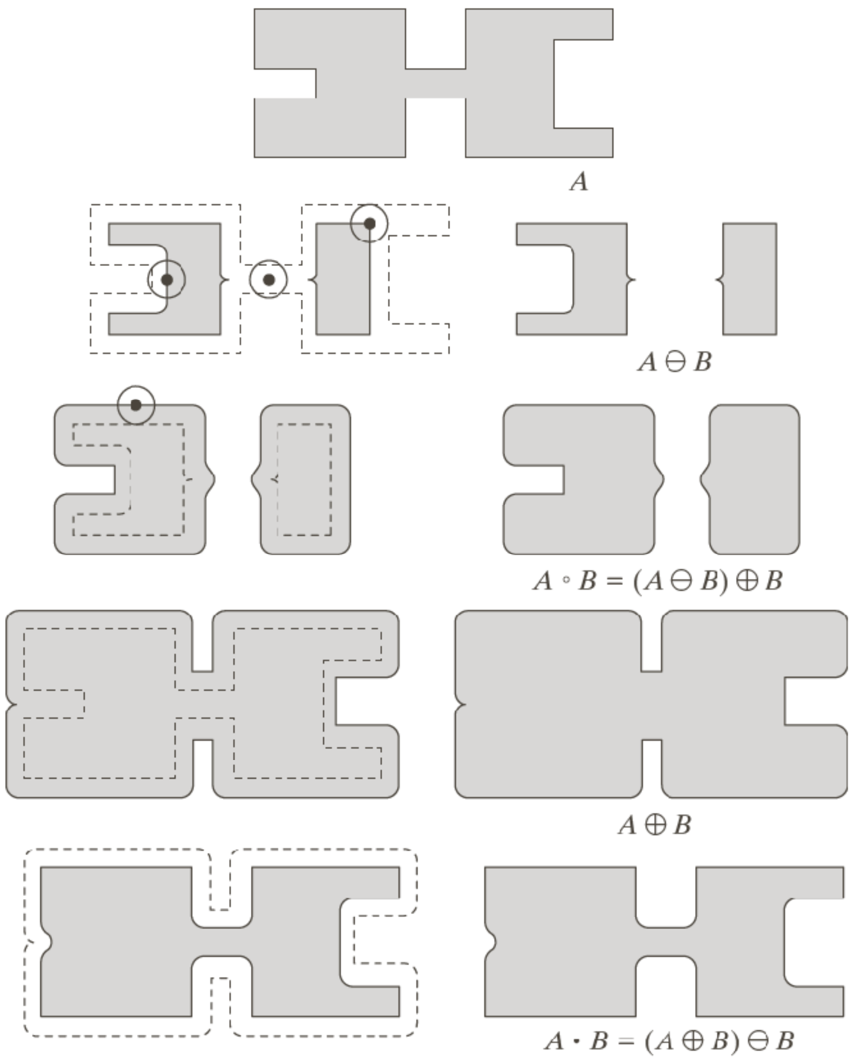
$$if \;\; C \subseteq D, \;\; then \; C \circ B \subseteq D \circ B$$

$$(A \circ B) \circ B = A \circ B$$

$$A \bullet B \subseteq A$$

$$if \;\; C \subseteq D, \;\; then \; C \bullet B \subseteq D \bullet B$$

$$(A \bullet B) \bullet B = A \bullet B$$



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

Construct filters by morphological operation

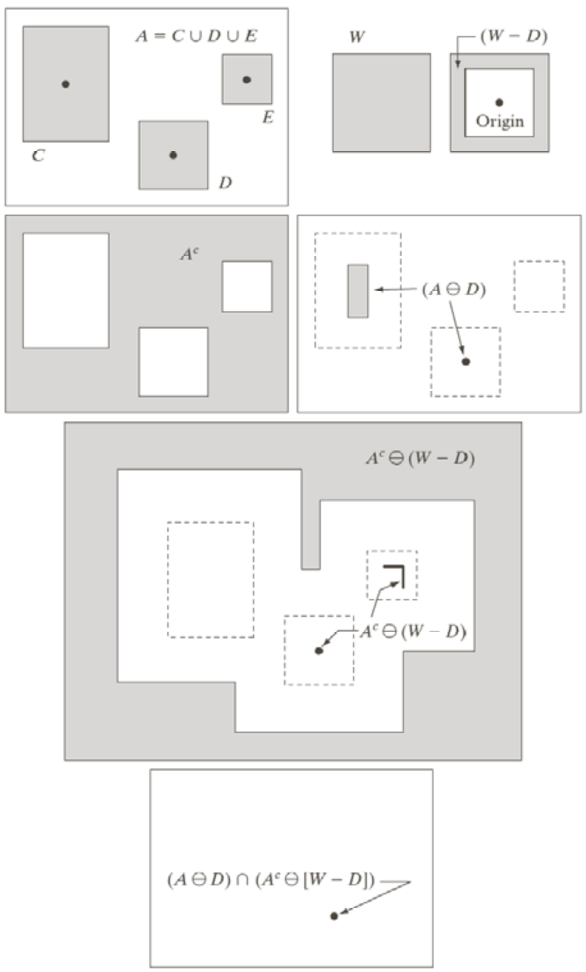
a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of *A*.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Hit or miss transformation

- A basic tool for shape detection

$$A * B = (A \ominus B) \cap [A^c \ominus (W - D)]$$



a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

Basic morphological algorithm

- Extracting boundaries, connected components, the convex hull, and skeleton of a region
- Boundary extraction: $\beta(A) = A - (A \ominus B)$

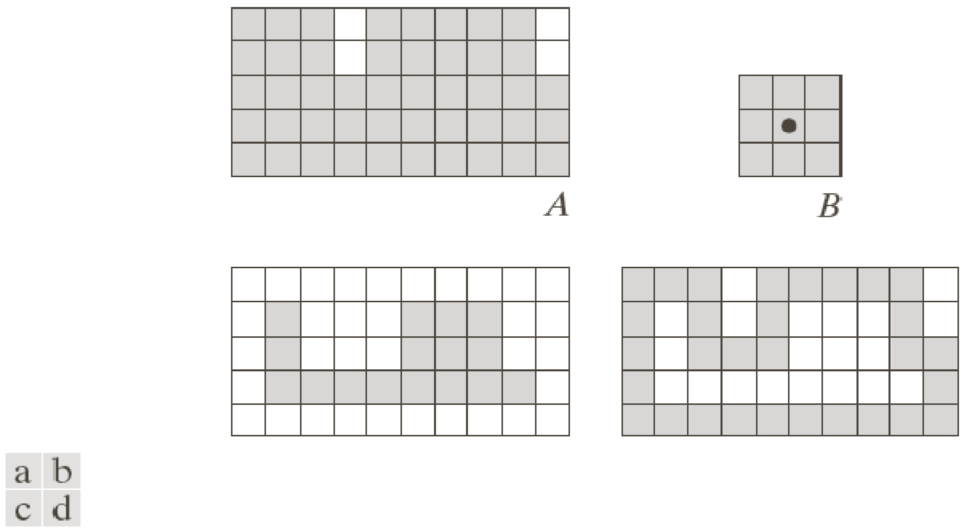


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

a b

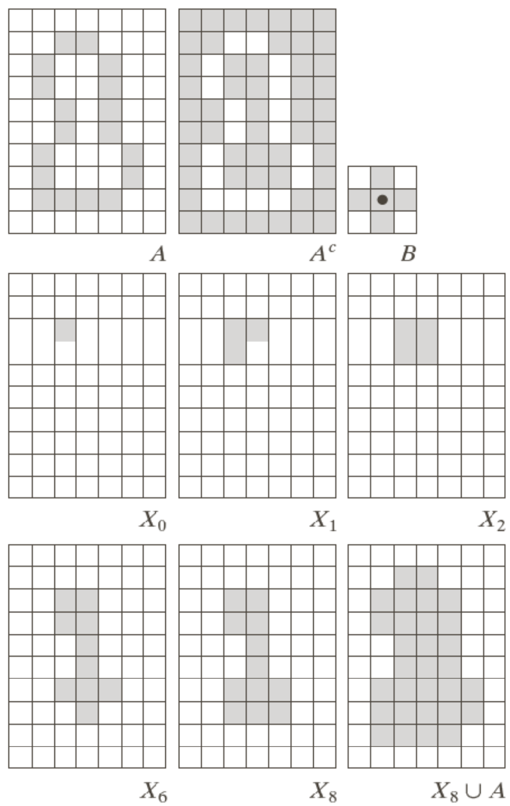
FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Hole filling

- Let X_0 be all 0's. Continue the following computing

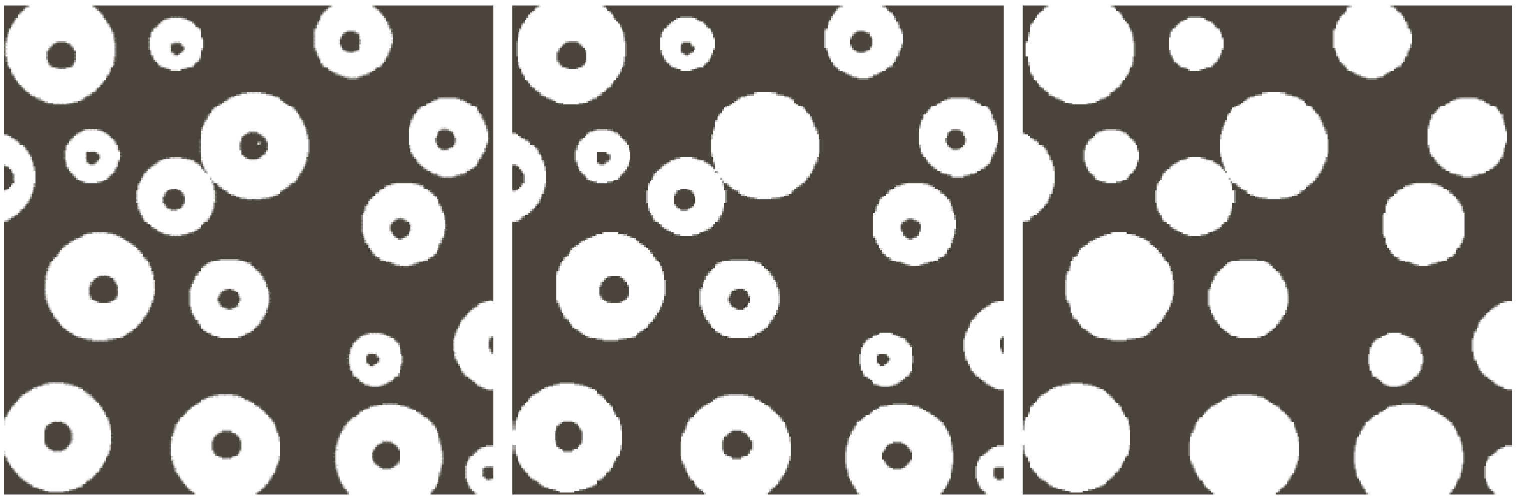
$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1, 2, 3, \ldots$$

until $X_k = X_{k-1}$



a	b	c
d	e	f
g	h	i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Extraction of connected components

- Let X_0 be all 0's. Continue the following computing

$$X_k = (X_{k-1} \oplus B) \cap A, k = 1, 2, 3, \ldots$$

until $X_k = X_{k-1}$

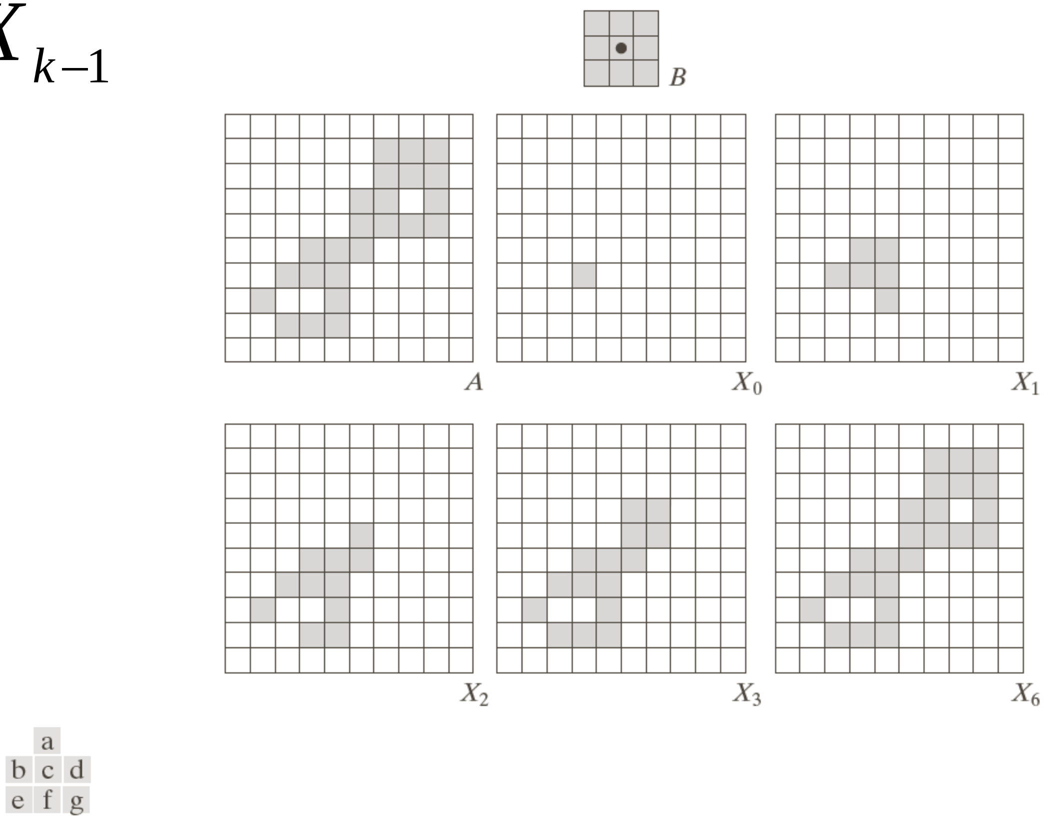
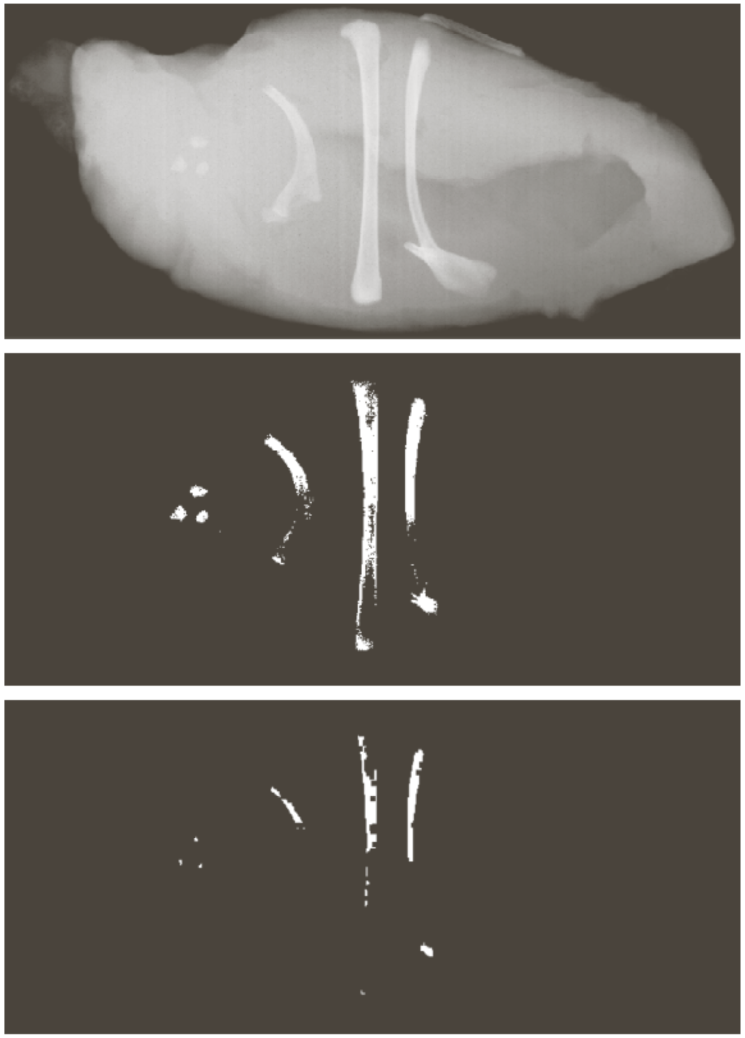


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex hull

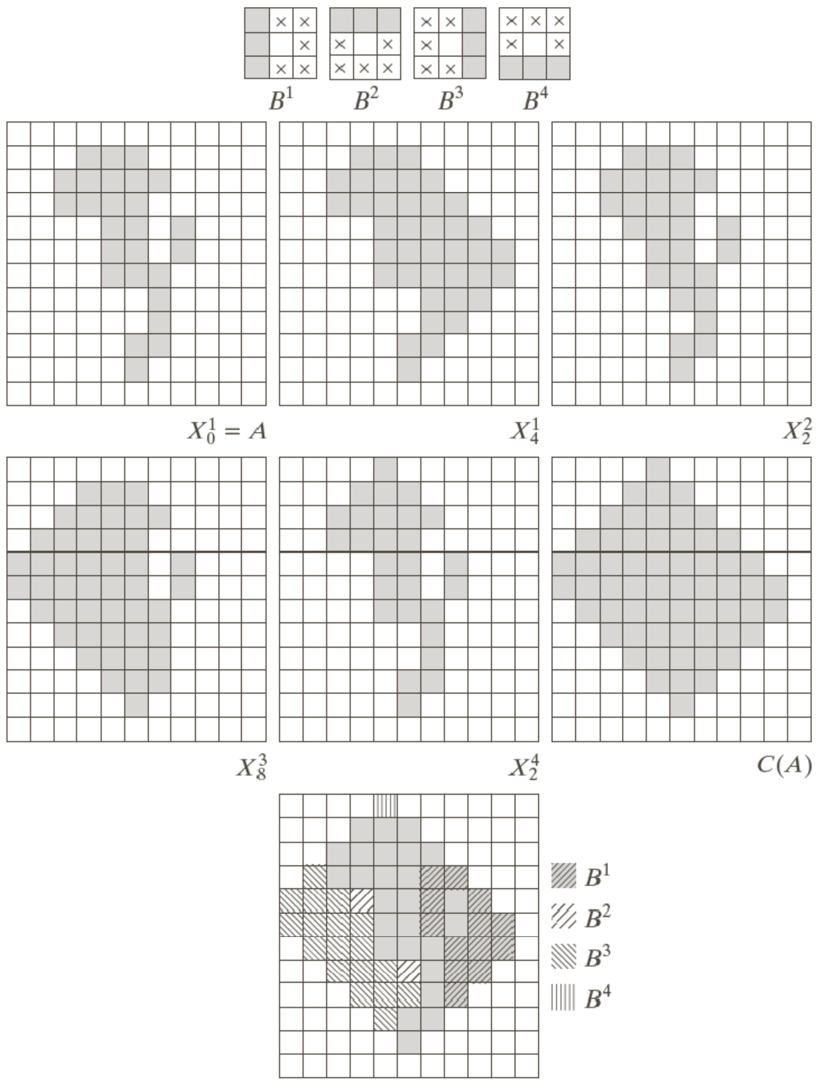
- Let X_0 be all 0's. Continue the following computing

$$X_0^i = A, i = 1, 2, 3, 4$$

$$X_k^i = (X_{k-1} * B^i) \bigcap A, i = 1, 2, 3, 4, k = 1, 2, 3...$$

until $X_k^i = X_{k-1}^i,\ \ then\ let\ D^i = X_k^i$

$$C(A) = \bigcup_{i=1}^{4} D^i$$



a
b c d
e f g
h

FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

Thinning and Thickening

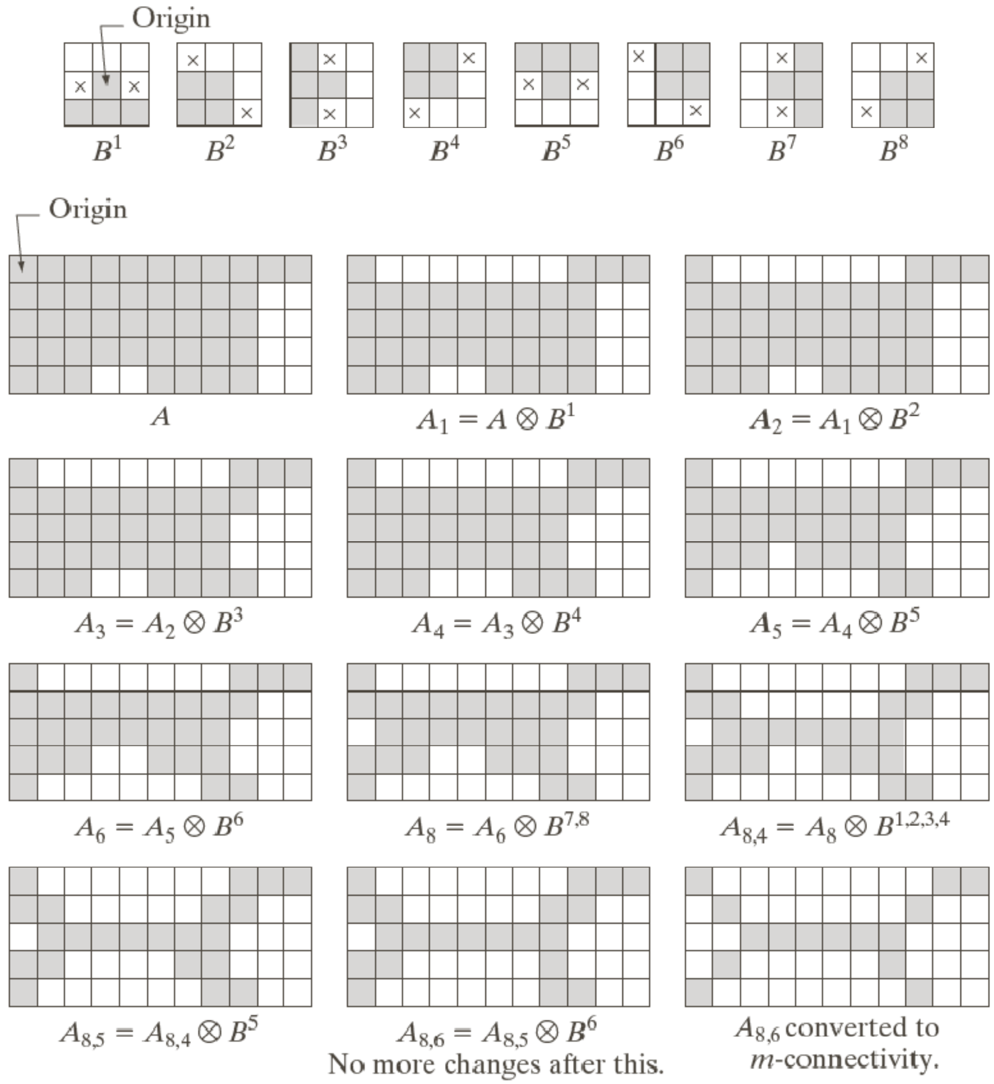
- The thinning of A by SE B

$$A \otimes B = A - (A * B) = A \cap (A * B)^c$$

- Thinning by a sequence of SE

$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

$$A \otimes \{B\} = ((..((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$



a		
b	c	d
e	f	g
h	i	j
k	l	m

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

Thickening

- The thickening of A by SE B

$$A \odot B = A \cup (A * B)$$

- Thickening by a sequence of SE

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

$$A \odot \{B\} = ((..((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

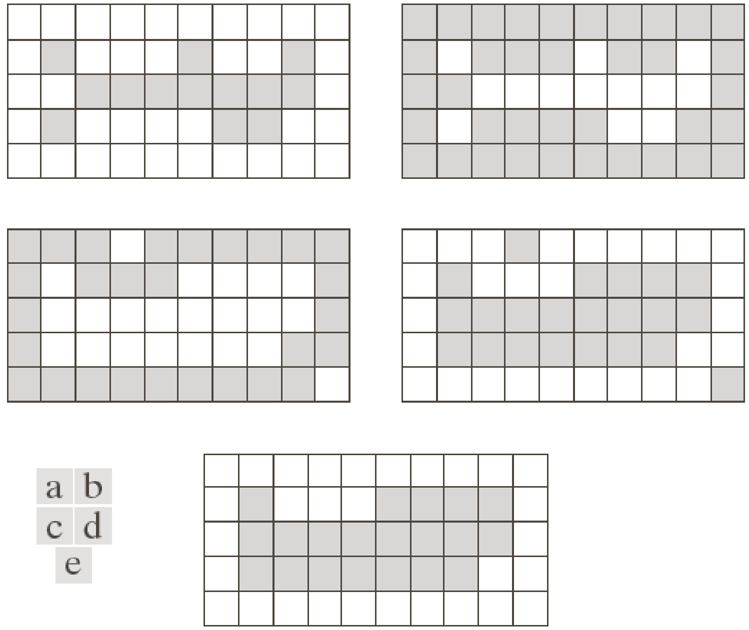


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

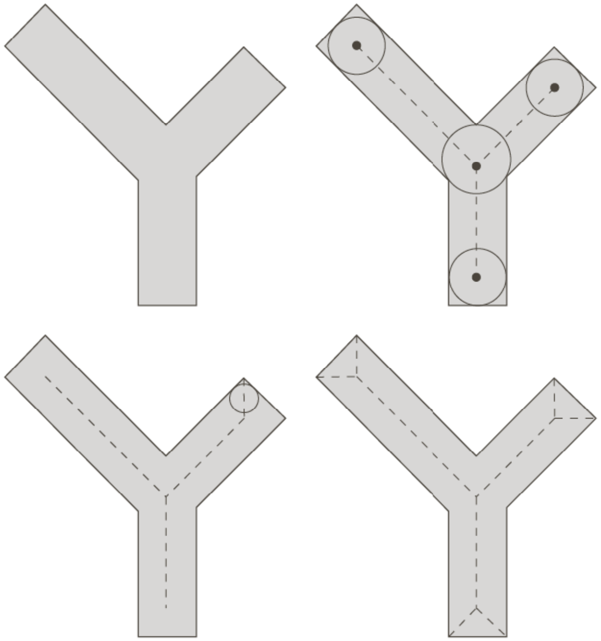
$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$$

$$(A \Theta kB) = ((\ldots((A \Theta B) \Theta B) \Theta \ldots) \Theta B)$$

$$K = \max\{k \mid (A \Theta kB) \neq \varnothing\}$$

a	b
c	d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.



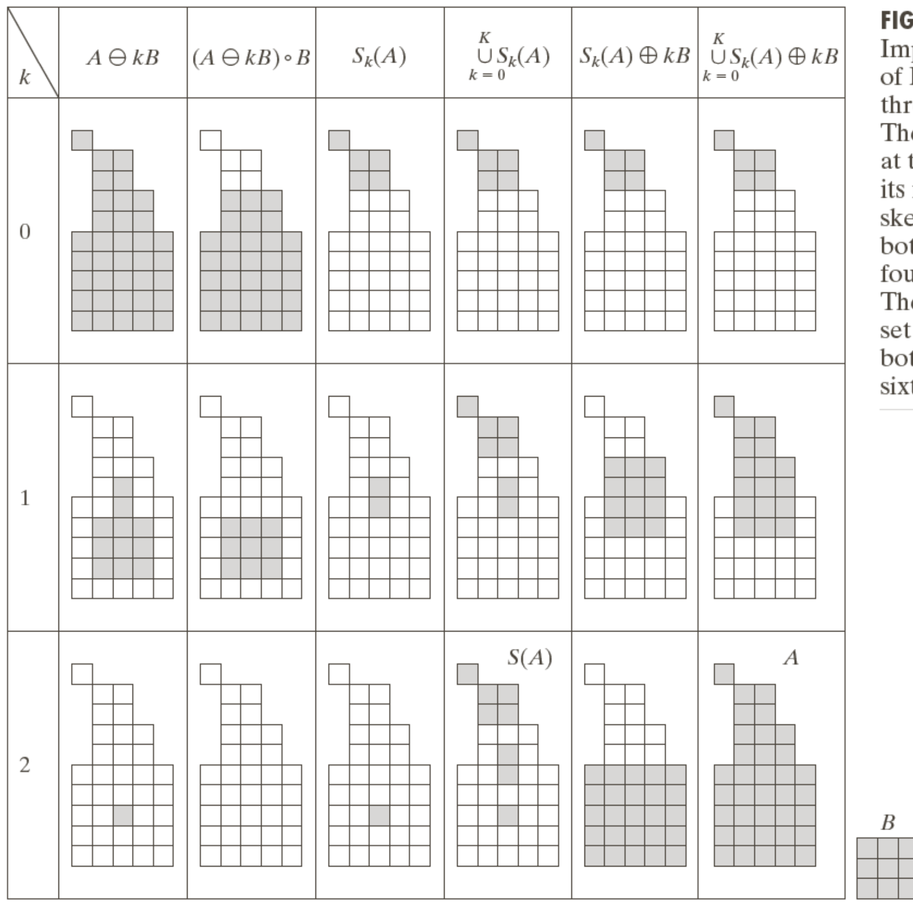


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

Pruning

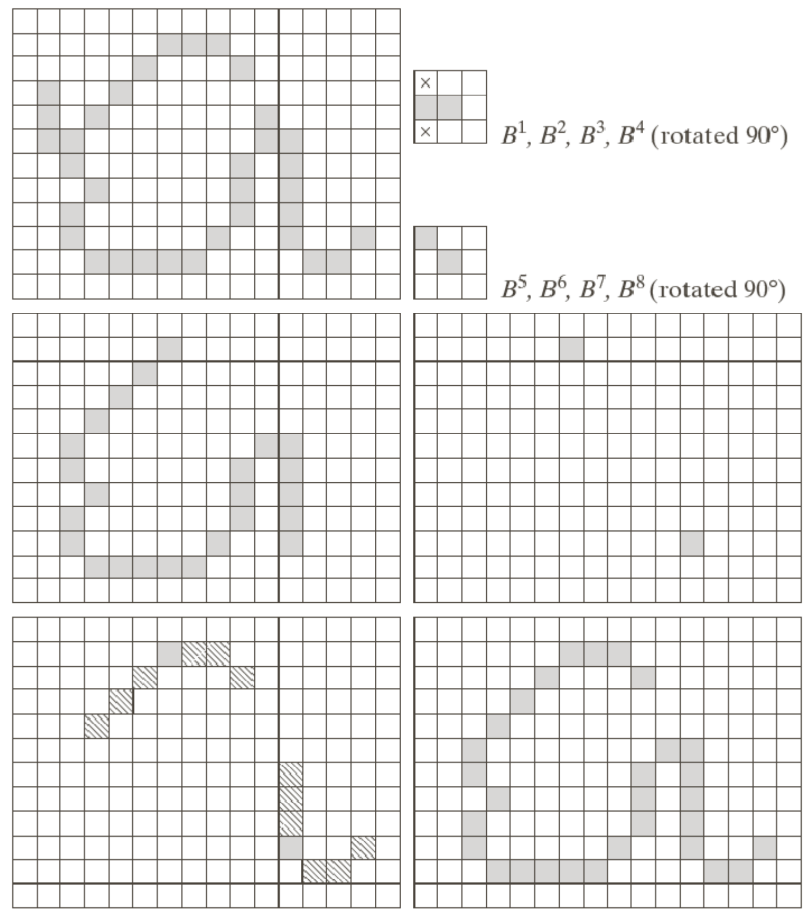
- Pruning is done by the following four steps

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 * B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$



a b
c
d e
f g

FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

Morphological Reconstruction

- Geodesic dilation

$$D_G^{(1)}(F) = (F \oplus B) \bigcap G$$

$$D_G^{(n)}(F) = D_G^{(1)}(F)[D_G^{(n-1)}(F)]$$

- Morphological reconstruction by dilation

$$R_G^D(F) = D_G^{(k)}(F), where\ D_G^{(k)}(F) = D_G^{(k-1)}(F)$$

- Geodesic erosion

$$E_G^{(1)}(F) = (F \Theta B) \bigcup G$$

$$E_G^{(n)}(F) = E_G^{(1)}(F)[E_G^{(n-1)}(F)]$$

- Morphological reconstruction by erosion

$$R_G^E(F) = E_G^{(k)}(F), where\ E_G^{(k)}(F) = E_G^{(k-1)}(F)$$

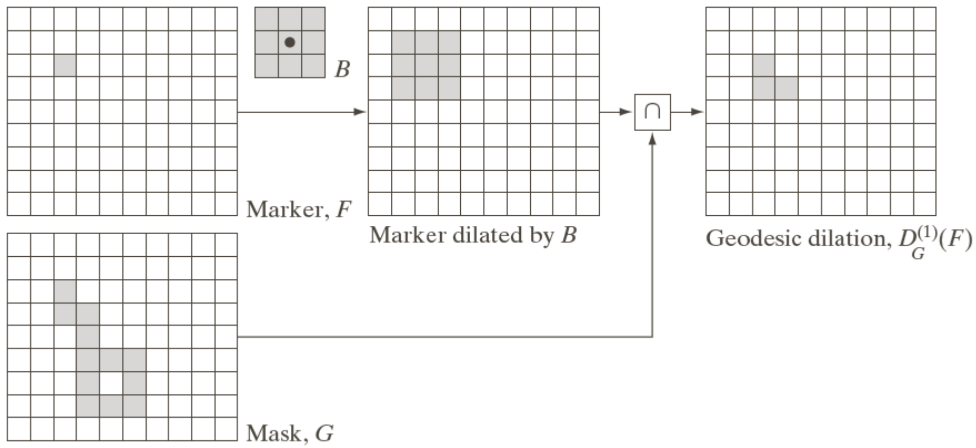


FIGURE 9.26
Illustration of geodesic dilation.

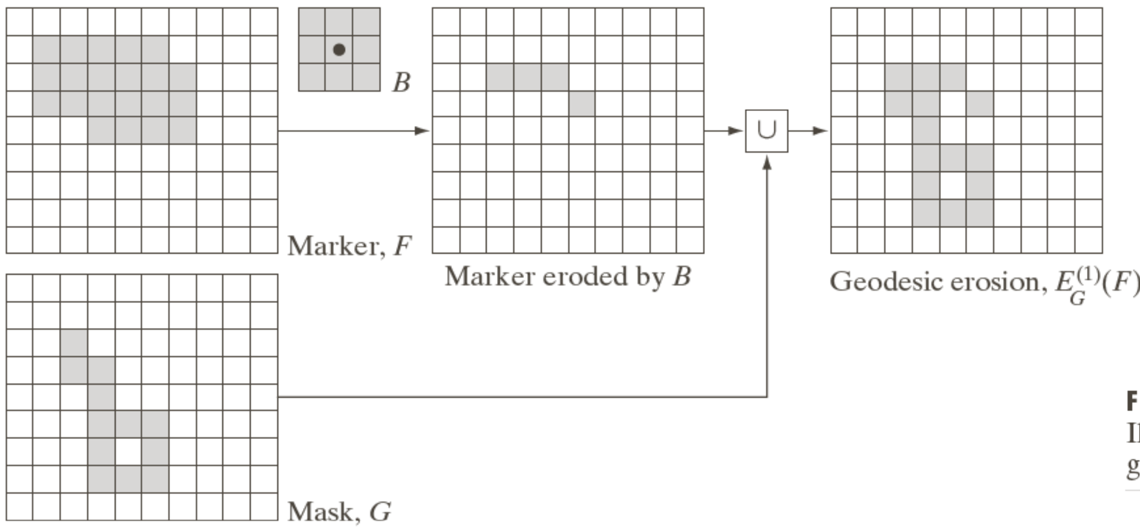
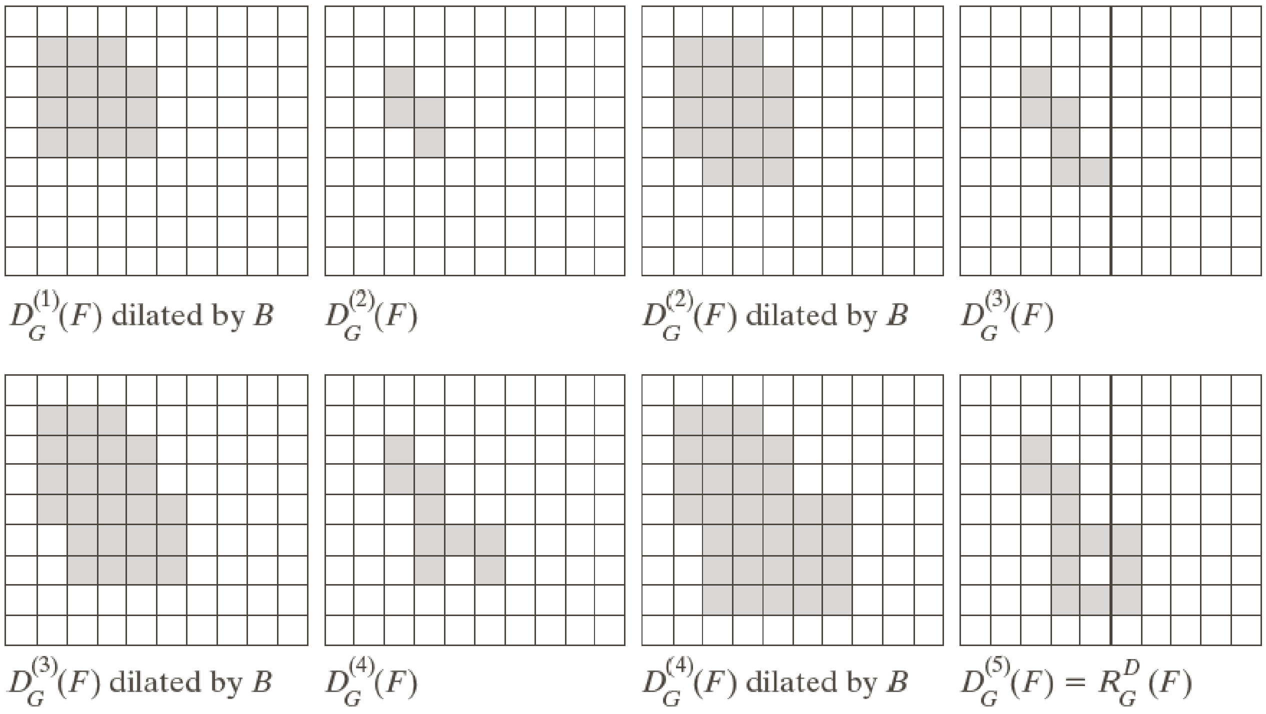


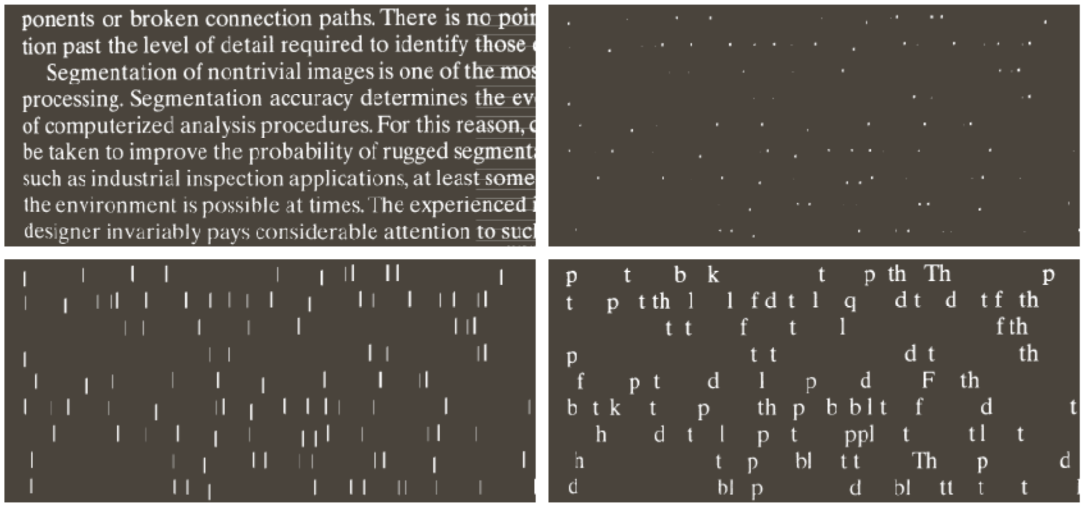
FIGURE 9.27 Illustration of geodesic erosion.



a b c d
e f g h

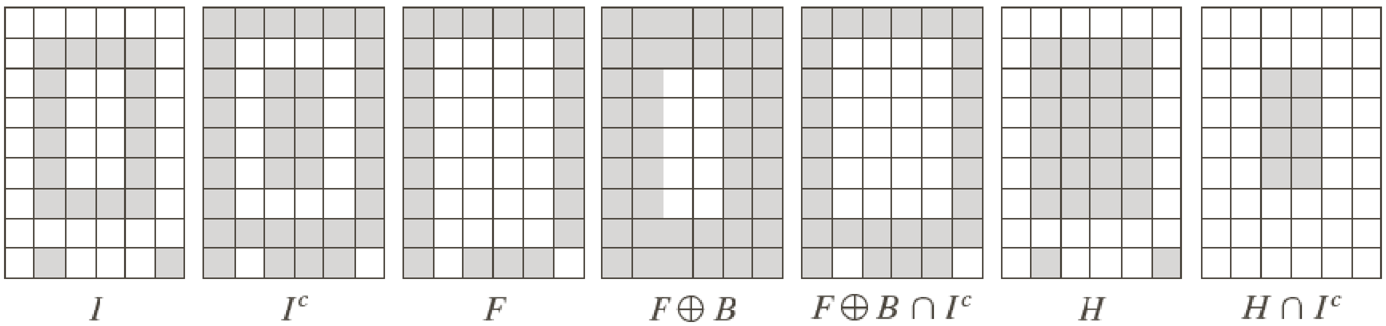
FIGURE 9.28 Illustration of morphological reconstruction by dilation. F, G, B and $D_G^{(1)}(F)$ are from Fig. 9.26.

Simple applications



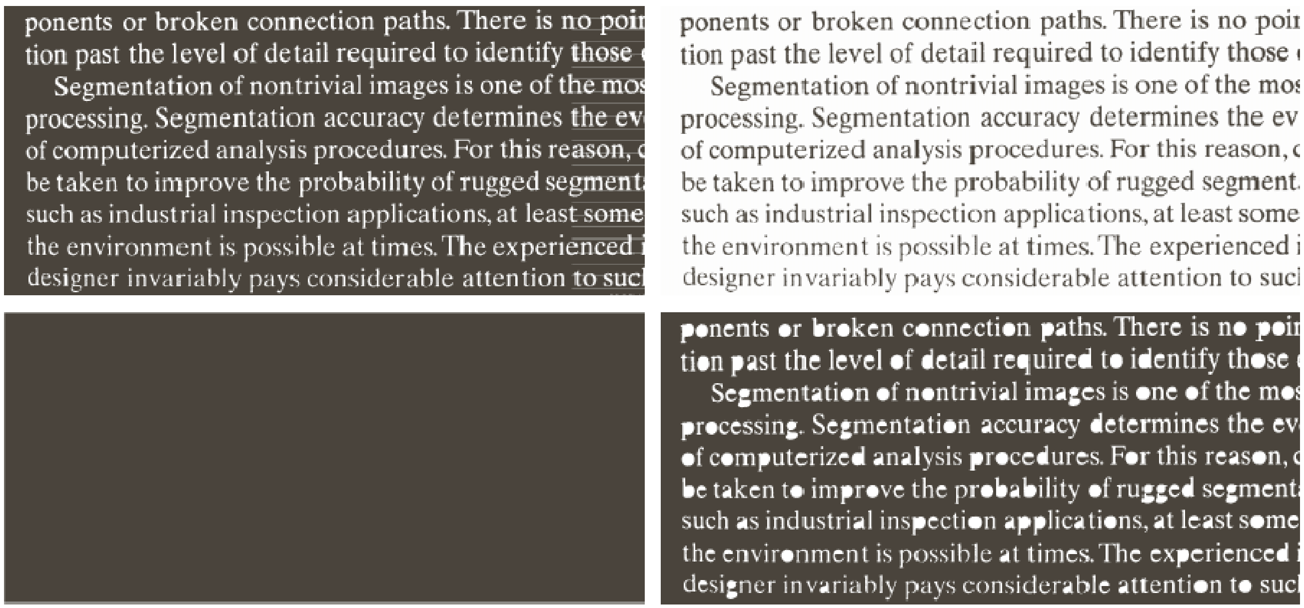
a b
c d

FIGURE 9.29 (a) Text image of size 918 × 2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51 × 1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.



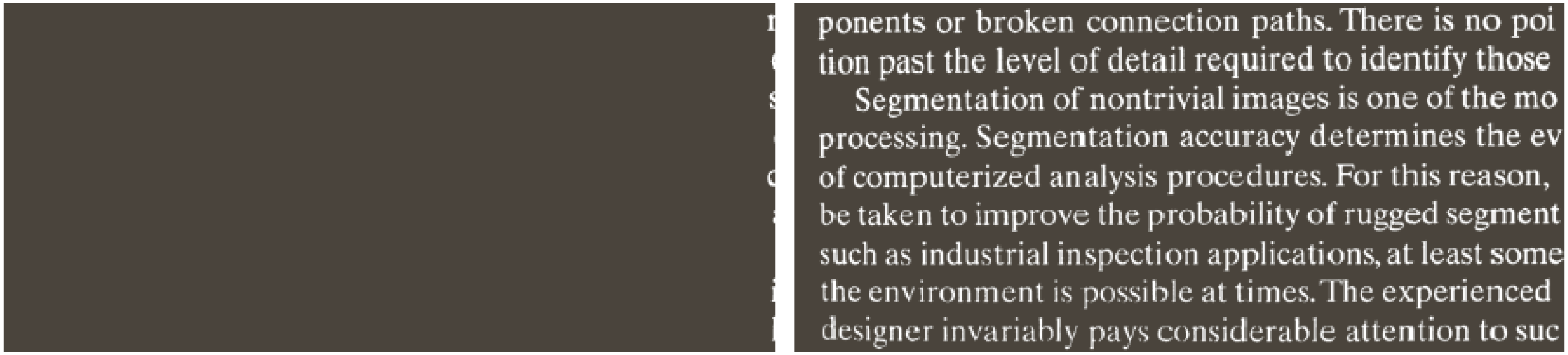
a b c d e f g

FIGURE 9.30 Illustration of hole filling on a simple image.



a b
c d

FIGURE 9.31 (a) Text image of size 918 × 2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).



a b

FIGURE 9.32 Border clearing. (a) Marker image. (b) Image with no objects touching the border. The original image is Fig. 9.29(a).

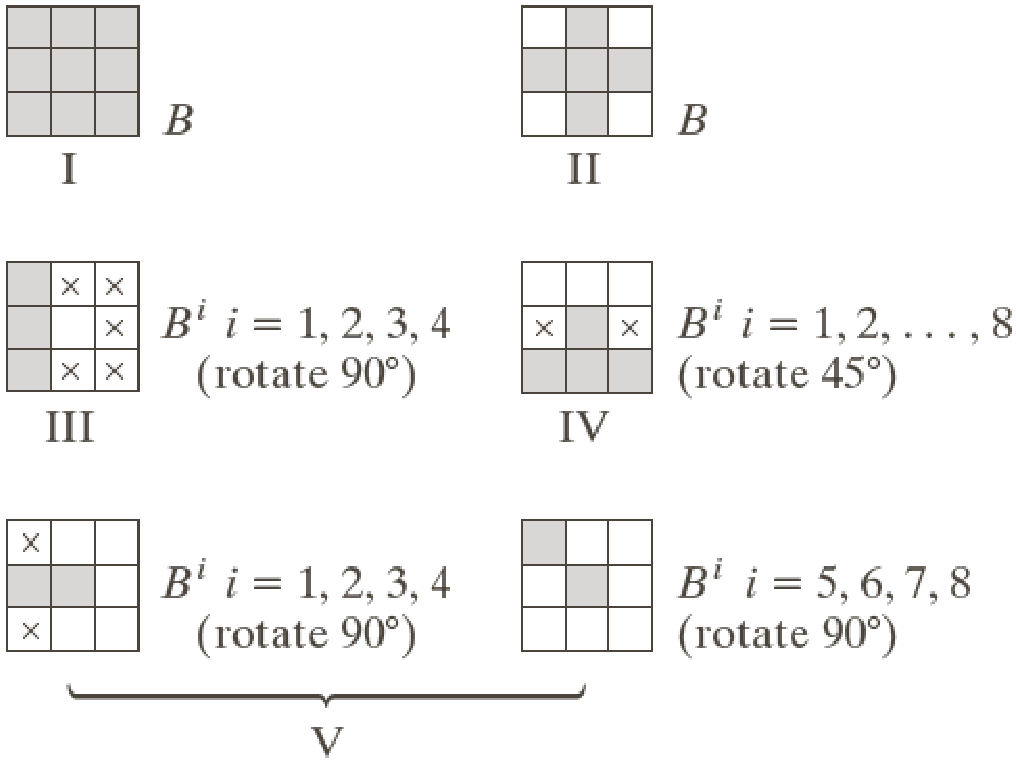


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

TABLE 9.1
Summary of morphological operations and their properties.

Operation	**Equation**	**Comments** (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w \mid w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \left\{z \mid (\hat{B}_z) \cap A \neq \varnothing\right\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \left\{z \mid (B)_z \subseteq A\right\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(*Continued*)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \ldots$	Fills holes in A; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \ldots$	Finds connected components in A; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \circledast B) = A \cap (A \circledast B)^c$; $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$; $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$; $A \odot \{B\} = ((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$; $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$; Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$; $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$; $X_3 = (X_2 \oplus H) \cap A$; $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and *mask* images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right]$; $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right]$; $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	k is such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B.
Hole filling	$H = \left[R_{I^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F.

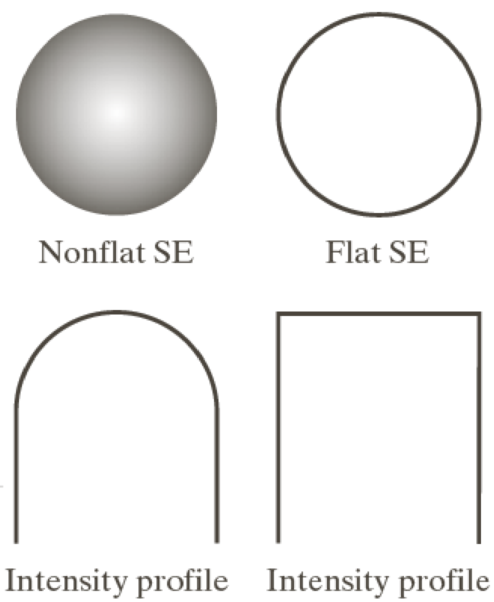
(*Continued*)

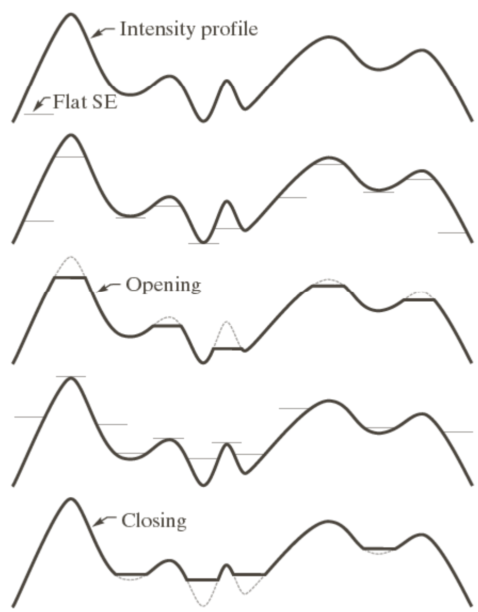
Gray-Scale Morphology

- Structure element
- Dilation $[f \oplus b](x, y) = \max_{(s,t)\in b}\{f(x-s, y-t)\}$
- Erosion $[f \Theta b](x, y) = \min_{(s,t)\in b}\{f(x+s, y+t)\}$
- Opening $f \circ b = (f \Theta b) \oplus b$
- Closing $f \bullet b = (f \oplus b) \Theta b$

a b
c d

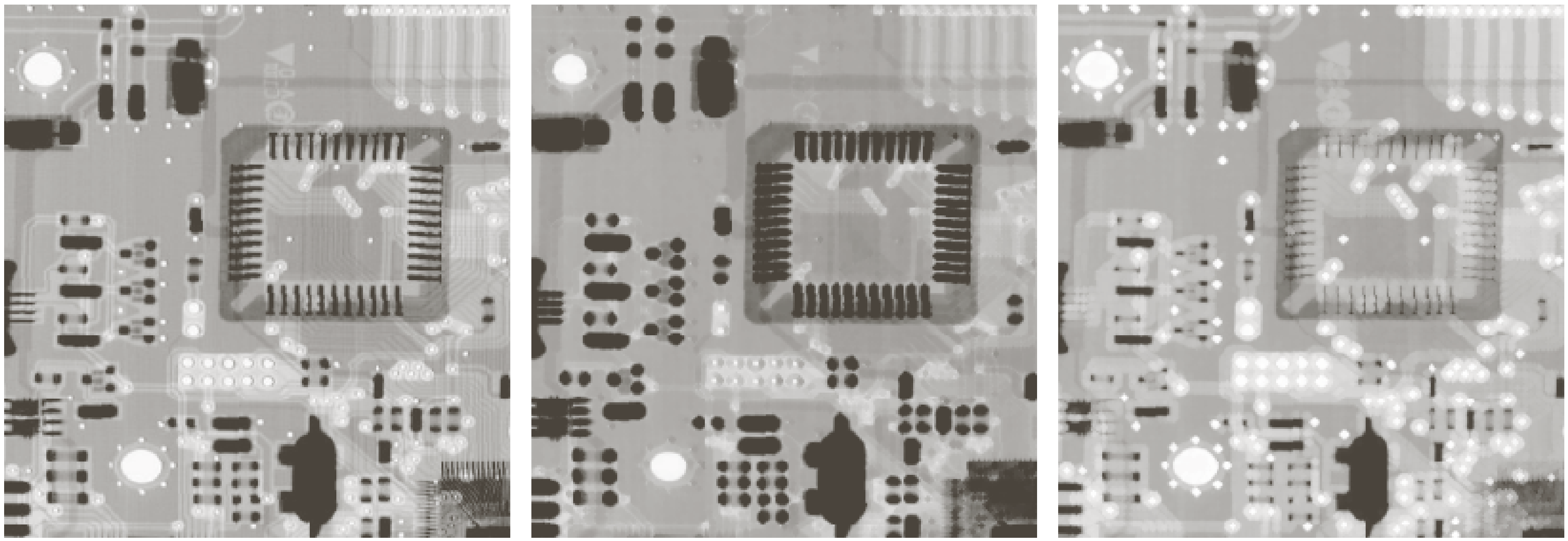
FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.





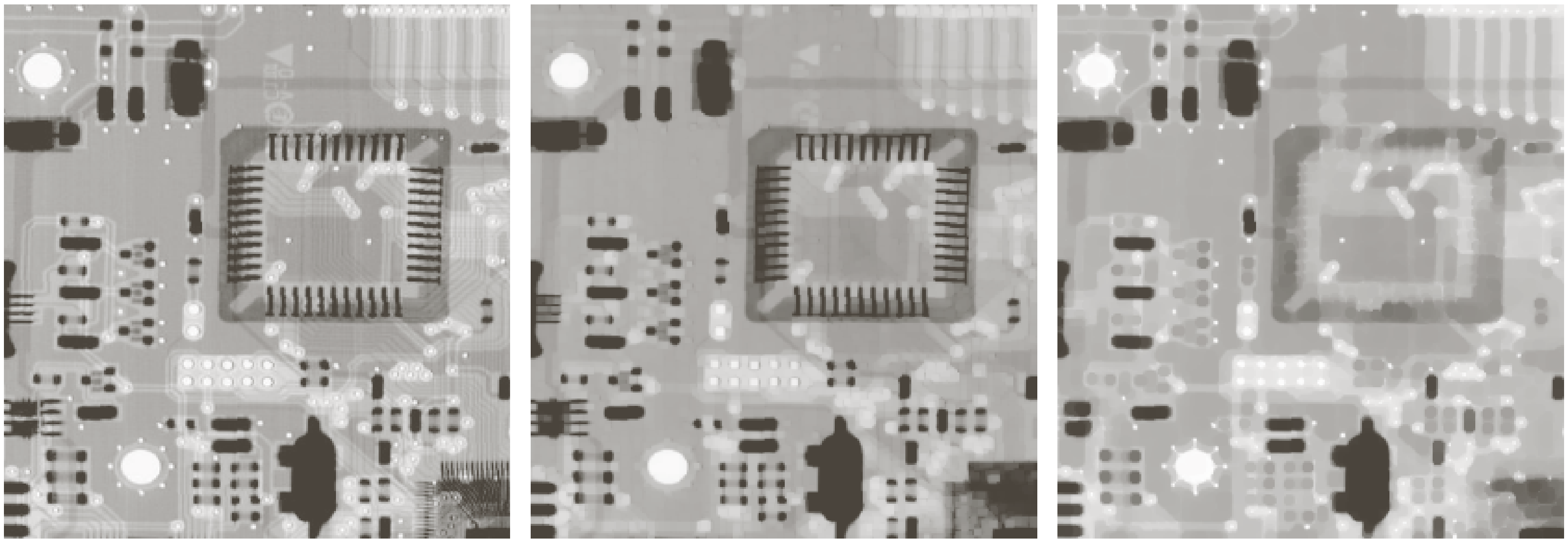
a
b
c
d
e

FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal.
(c) Opening.
(d) Flat structuring element pushed down along the top of the signal.
(e) Closing.



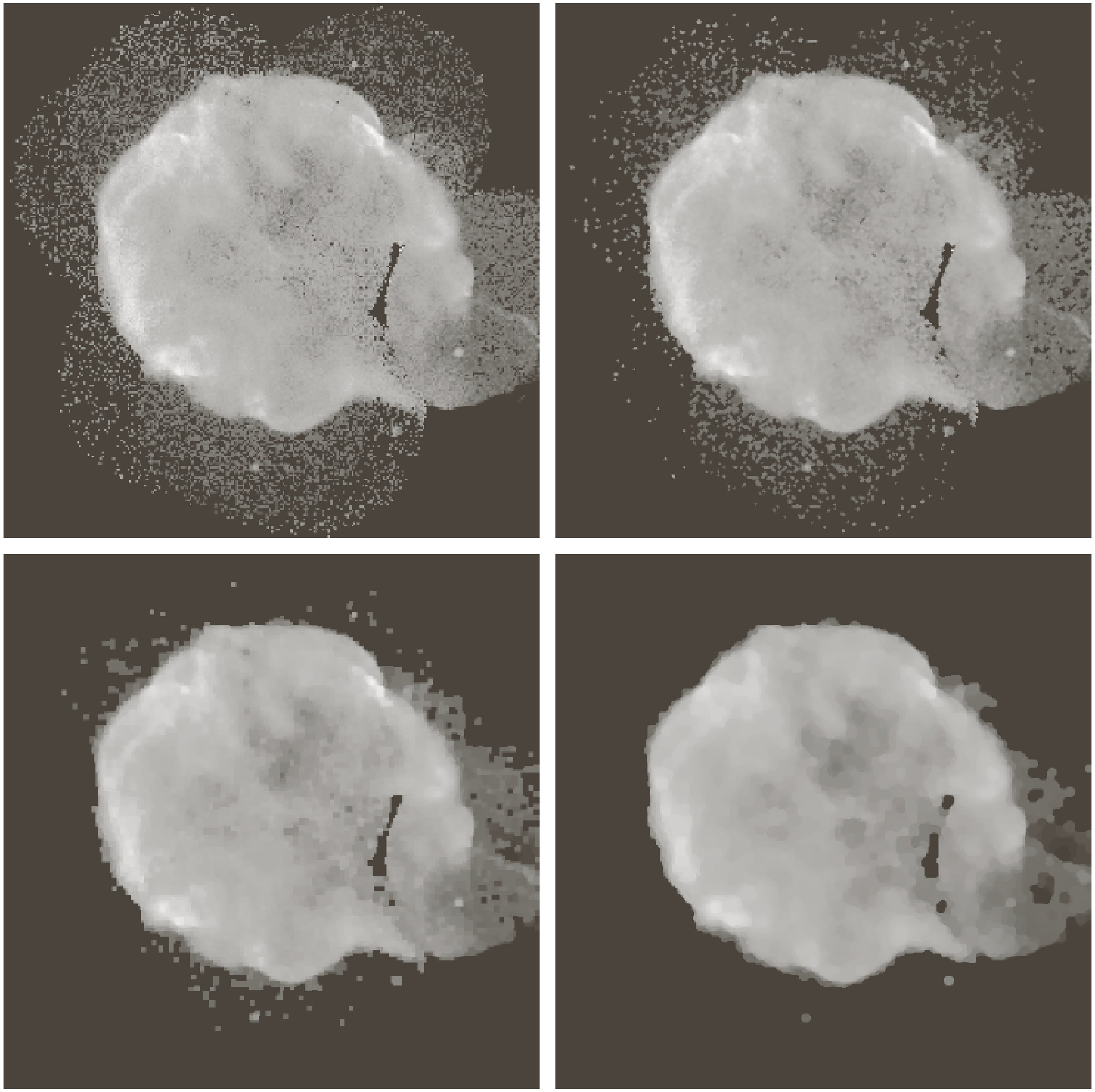
a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)



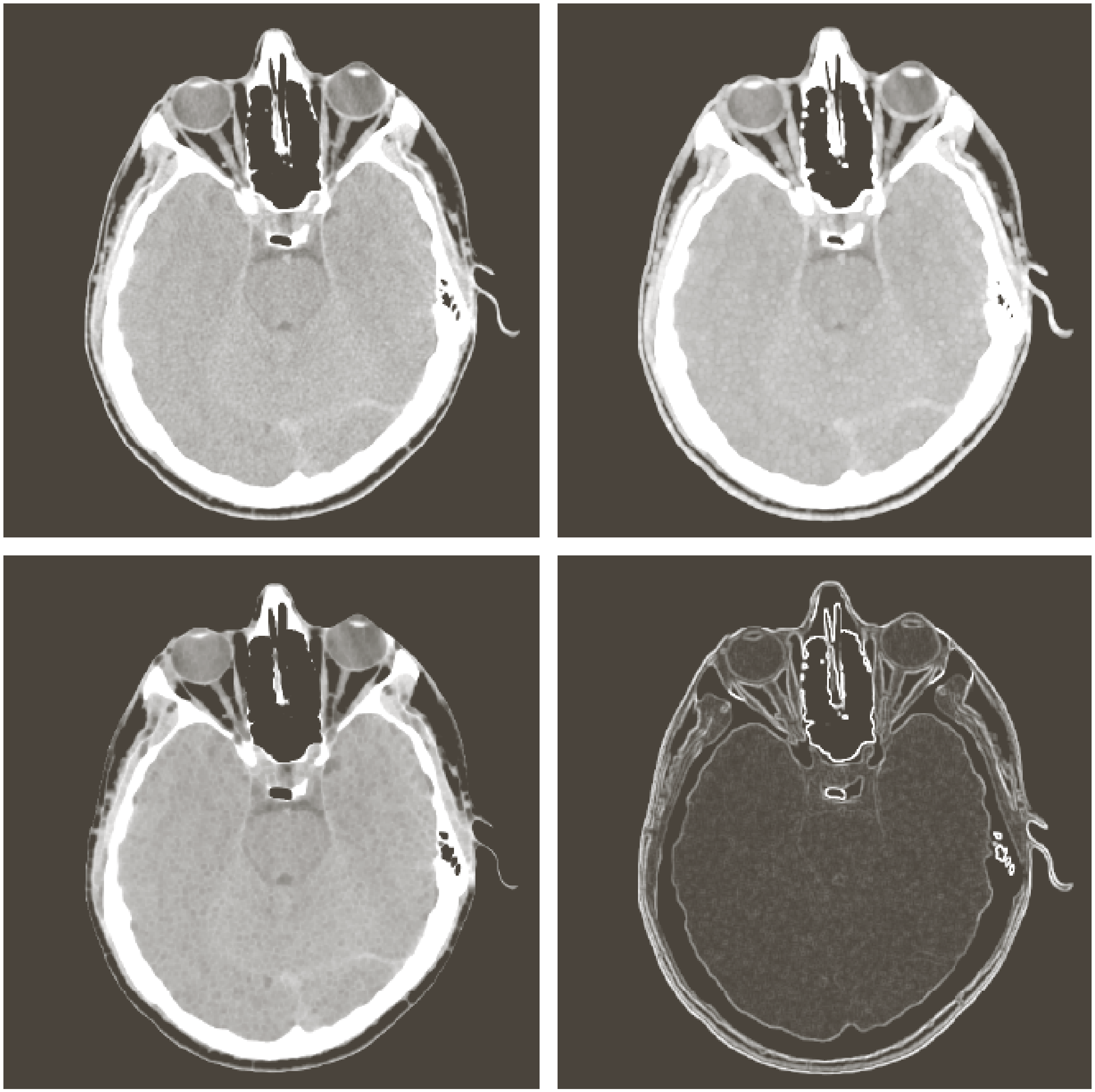
a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)



a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

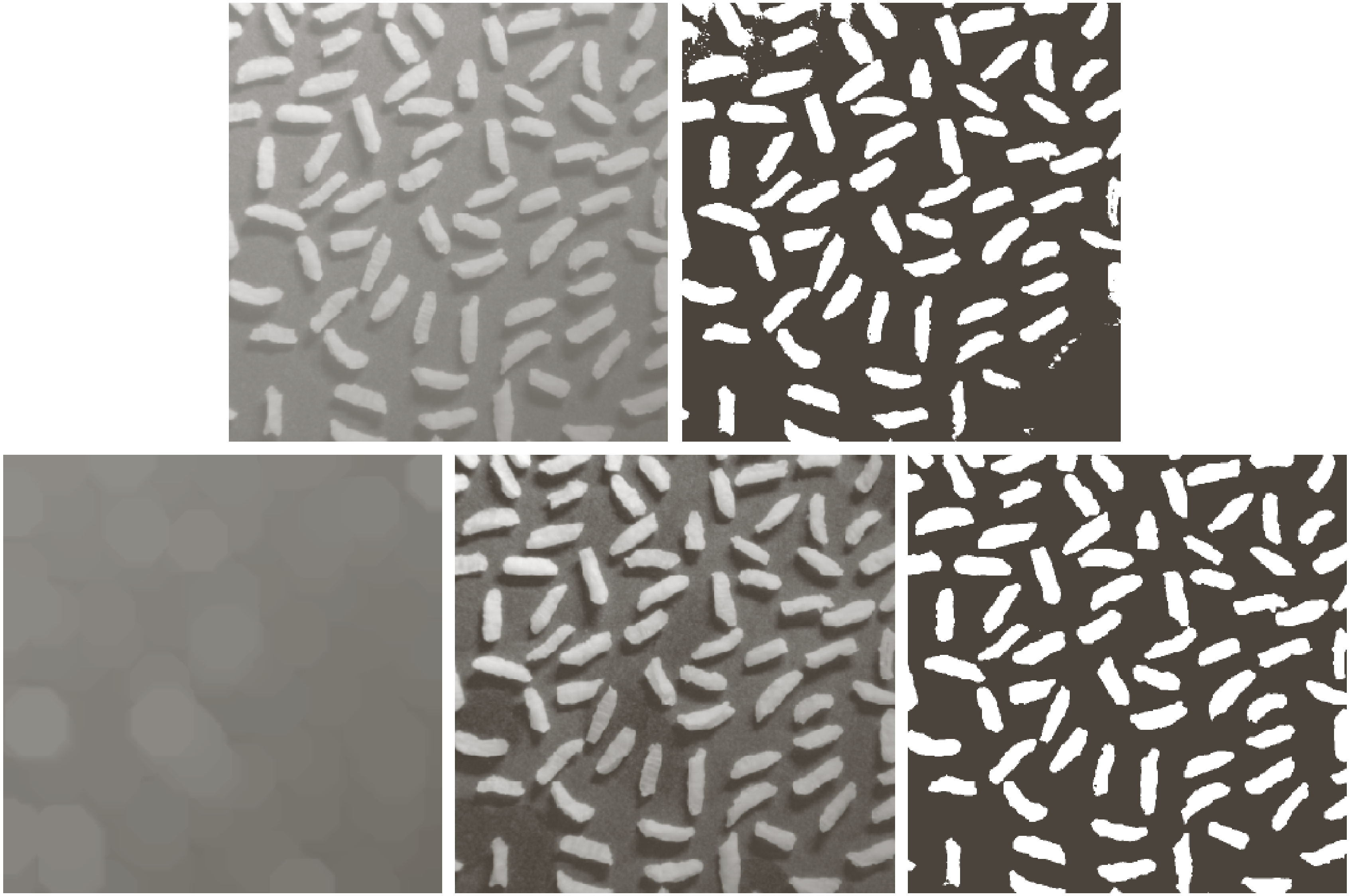
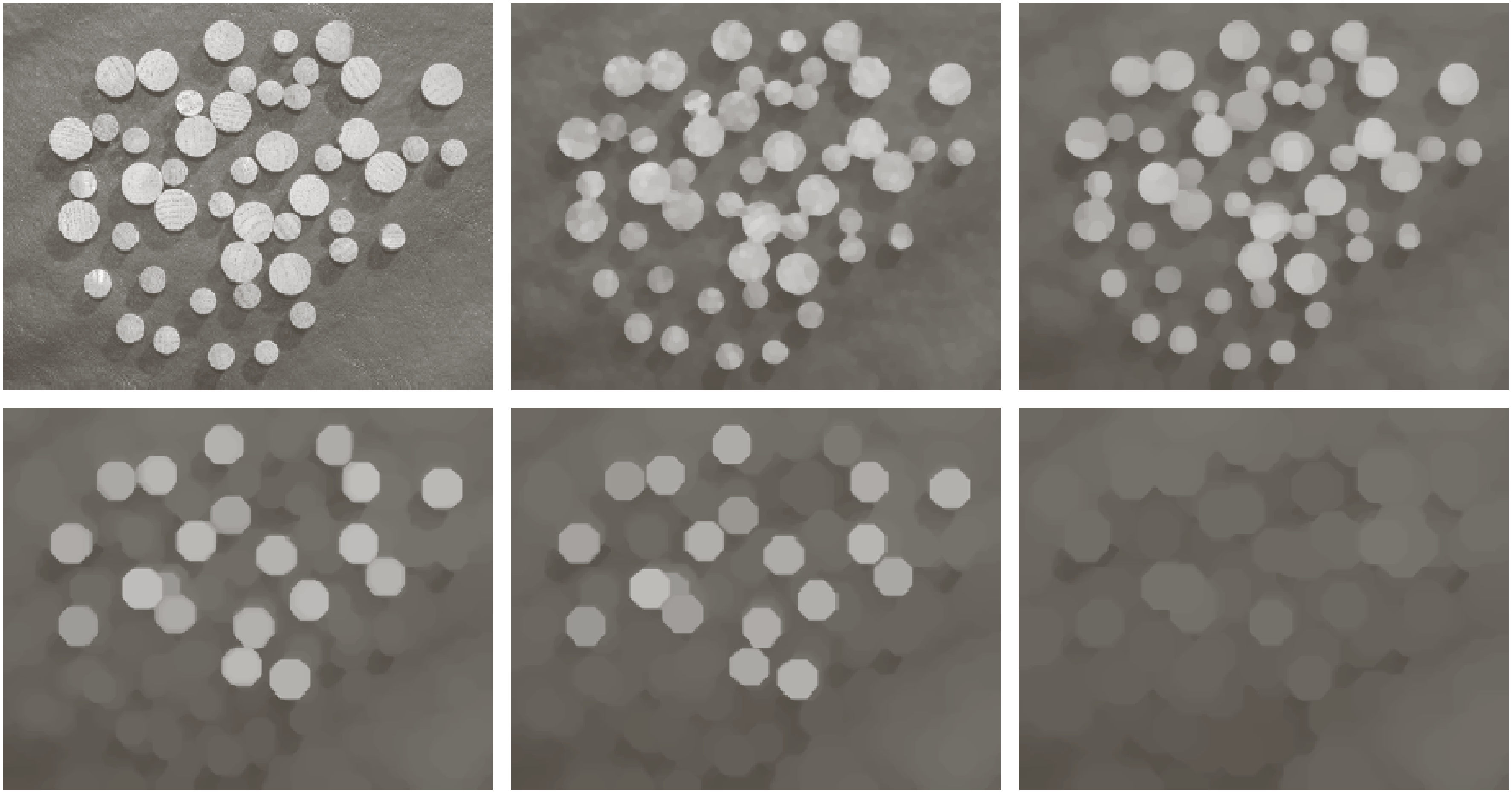
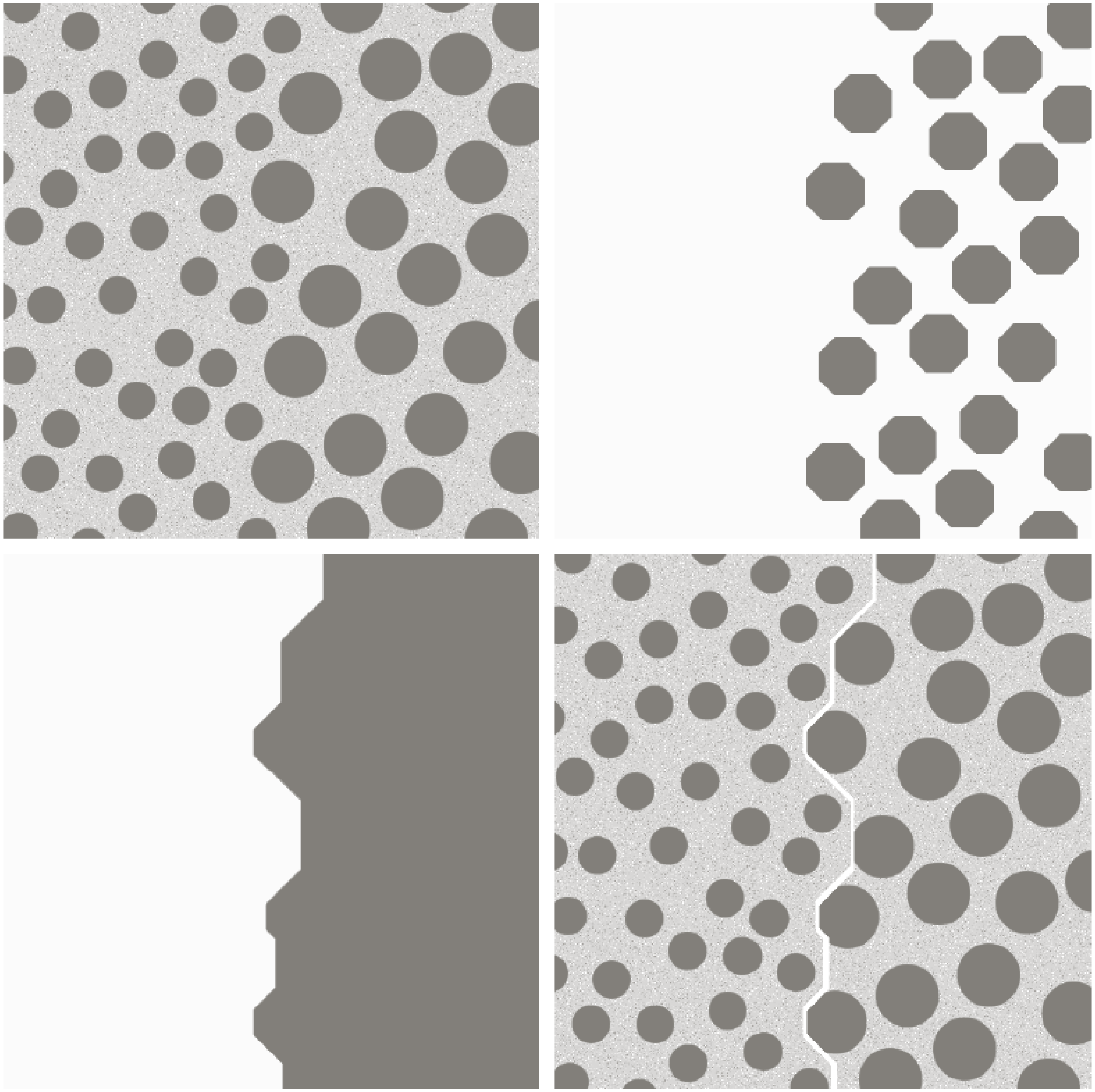


FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600×600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.



a	b	c
d	e	f

FIGURE 9.41 (a) 531 $\times$ 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)



a	b
c	d

FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

a	b	c
d	e	f
g	h	i

FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)