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Constructing the shaddow geometry of the new $W(23, 16)$

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A weighting matrix of length n and weight k denoted W(n,k) is an orthogonal $\{0, \pm 1\}$ matrix with rows of length $\sqrt{k}$. I would like to report on a new weighting matrix W(23,16), the smallest pair of numbers where the existense of the weighting matrix is currently not known. I will survey the details of the paper by the authors. This matrix was found in two stages: the one called finding the geometry which is equivalent to finding W modulu 2, and the whole matrix from the geometry called colouring. In this note we concentrate on the geometry stage.
Define [G] a (n, k) *shaddow geomtry* to be a set G of size n together with a family $\mathcal{L}$, $|\mathcal{L}| = n$, of susbstes called *lines* such that all lines have cardinality $n - k$, every two lines intersect in $n \mod 2$ points, and the dual statments. Define a shaddow matrix S for W, where each $\pm 1 \in$W is replaced by 0∈S and each 0∈W is replaced by 1∈S. In [G], each row of S is thought of as a line, and each column of S as a point, and S is the incidence matrix of the resulting shaddow geometry. For $(n, k) = (23, 16)$, we show that more that 50% of the pairs intersect at a single point. Therefore, finding a $(23, 16)$ geometry came as a surprise.
A local shaddow geomtry is given by a rectangular incidence matrix, satisfying an axiom system which is a natural restriction of the axiom system for the (full) shaddow geometry. Singling out a line ℓ in a shaddow geometry and restricting to the points of that line, one obtaines a local geometry. Let z_i be the number of lines $\Updownarrow$ with $|\Updownarrow \cap \ell| = i$. Then one can obtain linear equations for the $\{z_i\}$. For $(n, k) = (23, 16)$ there are 14 possible 4-tuples (z_7, z_5, z_3, z_1), from which we can study the set of all local geometries. We worked with a local geometry of the type $(z_7, z_5, z_3, z_1) = (2, 0, 4, 16)$ and extended it to a full geometry. The core of our construction is a 16×16 $\{0, 1\}$-matrix, enveloped by that local geometry and its dual. The 16×16 matrix has a 4×4 block structure with blocks coming from a small family of 512 blocks. We used linear algebra modulo 2 and branch cutting to find the complete geometry.

References

[G] A. Goldberger, On the Finite Geometry of $W(23, 16)$, http://arxiv.org/abs/1507.02063