Usage of (multiple) code invariant to find the symmetric $W(23,16)$

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Hadamard and weighing matrices $W(n,w), 1 \leq w \leq n$, have applications in communications, chemistry and quantum computation.
Given two matrices A, B, they are Hadamard equivalent if B is obtained from A by permuting rows or columns or by multiplying rows or columns by -1.
Given (n,w), the number of Hadamrd equivalence classes of $W(n,w)$ in an open problem. In particular the question if this number is different than zero (existence) is an open question.
Another open problem is given (n,w) to find $W(n,w)$ which is (anti)symmetric.
Given a matrix R we define $ci(R)$, the code invariant of R. R and S are Hadamard equivalent if and only if $ci(R) = ci(S)$. However the complexity to calculate $ci(R)$ increases with the length and with the width of R, and $ci(R)$ becomes uncomputable for large R.
Therefore we proceed to define the multiple code invariant $mci(R)$ of a matrix R. It is a weaker invariant based on all rectangular submatrices of R. If R and S are Hadamard equivalent then $mci(R) = mci(S)$. Thus $mci(R) \neq mci(S)$ implies that R and S are not Hadamard equivalent.
Given that $mci(R) = mci(S)$ we have, under mild restrictions, an efficient algorithm that either finds a Hadamard equivalence between them or determines that R and S are not Hadamard equivalent.
Therefore we are able to find (under those restrictions) $Aut(R)$, the group of all Hadamard equivalences of R.
After $Aut(R)$ is calculated, it is used to find if there exists S which is equivalent to R and is (anti)symmetric.
We were able to implement the above procedure to the $W(23,16)$ that was found using the shadow geometry method and to find a symmetric $W(23,16)$. We also applied this method to the matrices $W(n,16), n = 23, 25, 27, 29$ that were found using the tiling method and found that there is not an (anti)symmetric W that is Hadamard equivalent any of them.

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